# MAT 215A, Winter 2023
# Homework 3

**Due before 10:00 on Wednesday, February 8**

*Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.*

**1.** Consider $S^1$ as the unit circle in $\mathbb{C}$, then the map $f(z) = z^m$ defined a continuous map from $S^1$ to $S^1$. Find its degree (that is, class in $\pi_1(S^1)$).

**2.** Let $v(x, y)$ be a vector field on a plane which does not vanish on the unit circle. The *index* of $v$ is the degree of the map $\frac{v}{|v|} : S^1 \to S^1$. Find the degrees of the following vector fields: (a) $v(x, y) = (x, y)$ (b) $v(x, y) = (y, -x)$ (c) $v(x, y) = (y, x)$ (d) $v(x, y) = (x^2 - y^2, 2xy)$.

**3.** a) Let $f : X \to Y$ be a continuous map which sends the basepoint $x_0$ to the basepoint $f(x_0) = y_0$. Prove that it defines a group homomorphism $f_* : \pi_1(X) \to \pi_(Y)$ which sends a loop $\gamma : [0, 1] \to X$ to $f \circ \gamma : [0, 1] \to Y$.

b) Assume that $f, g : X \to Y$ are homotopic. Prove that the corresponding maps of fundamental groups coincide.

**4.** Use problem 3 to prove that homotopy equivalent spaces have isomorphic fundamental groups.