MAT 215A, Winter 2023
Homework 4

Due before 10:00 on Monday, February 27

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. In this problem we use the presentation of the two-torus as $\mathbb{R}^2/\mathbb{Z}^2$.
a) Prove that any matrix $A \in \mathrm{SL}(2, \mathbb{Z})$ defines a homeomorphism φ_A from the two-torus to itself.
b) Let M_1, M_2 be two solid tori, define the 3-manifold

$$M = \frac{M_1 \sqcup M_2}{(p \sim \varphi_A(p),\ p \in \partial M_1, \varphi_A(p) \in \partial M_2)}$$

In other words, we glue M_1 with M_2 by identifying their boundaries along the map φ_A. Compute the fundamental group of M using Seifert-van Kampen theorem.

2. Compute the fundamental group of the complement to n points in $\mathbb{R}^2$.

3. Compute the fundamental group of the complement to n points in $\mathbb{R}^3$.

4. Let p_1, p_2, p_3, p_4 be the vertices of a regular tetrahedron in $\mathbb{R}^3$ and let p_5 be the center of this tetrahedron. Let X be the CW complex obtained as a union of all possible triangles with vertices at p_i: 4 faces of the tetrahedron and 6 triangles connecting the edges of the tetrahedron with p_5. Compute $\pi_1(X)$.