MAT 215A, Winter 2023
Homework 5

Due before 10:00 on Monday, March 6

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. Suppose that $(g_1 - 1)$ divides $(g_2 - 1)$ and $g_1, g_2 \geq 1$. Prove that there is a covering $\Sigma_{g_2} \to \Sigma_{g_1}$ where $\Sigma_{g_1}, \Sigma_{g_2}$ are genus g_1 and g_2 surfaces respectively.

2. Let A be a 2×2 integer matrix and $\det(A) \neq 0$. Prove that it defines a covering map $T^2 \to T^2$ and compute its degree.

3. a) Describe all subgroups of $\pi_1(T^2)$ (you can use any results from algebra without proof, provided that you state them correctly).
b) For each subgroup H in (a) there is a covering space $p : X_H \to T^2$ such that

$$p_* \pi_1(X_H) = H.$$

Describe X_H explicitly.

4. a) Classify all connected covering spaces $X \to \mathbb{RP}^n$ up to homeomorphism for $n \geq 2$.
b) Classify all connected covering spaces $X \to \mathbb{CP}^n$ up to homeomorphism for $n \geq 1$.