MAT 215C, Spring 2021
Homework 1

Due before 3:10 on Monday, April 5

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. Let $f : U \to V$ be a linear map between finite-dimensional vector spaces, $f^* : V^* \to U^*$ is the dual map. Prove that
(a) $\operatorname{Ann}(\operatorname{Im} f) = \operatorname{Ker} f^*$ (b) $\operatorname{Ann}(\operatorname{Ker} f) = \operatorname{Im} f^*$.

2. Let

$$C_* = [\ldots \xrightarrow{\partial} C_{i+1} \xrightarrow{\partial} C_i \xrightarrow{\partial} C_{i-1} \xrightarrow{\partial} \ldots]$$

be a chain complex of finite-dimensional vector spaces, and let

$$C^* = [\ldots \xrightarrow{\delta} C^{i-1} \xrightarrow{\delta} C^i \xrightarrow{\delta} C^{i+1} \xrightarrow{\delta} \ldots]$$

be the dual complex. Use problem 1 to prove that $\dim H^i(C^*, \delta) = \dim H_i(C_*, \partial)$. *Hint: compute the dimensions of kernels and images of δ in terms of the ones for ∂.*

3. Compute the simplicial homology and cohomology of the letter A considered as a graph with 5 vertices and 5 edges.

4. Compute the cellular homology and cohomology of (a) S^2 (b) S^n (c) $\mathbb{RP}^2$ with integer coefficients.