MAT 215C, Spring 2021 Homework 2

Due before 3:10 on Monday, April 12

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. Compute the cup product in cohomology of (a) S^2 (b) S^n (c) \mathbb{RP}^2 with integer coefficients.

2. Given the cup product in $H^*(X)$ and $H^*(Y)$, describe the cup product for the disjoint union $H^*(X \sqcup Y)$.

3. Use Künneth formula and problem 1 to describe the cup product in cohomology of (a) $S^2 \times S^2$ (b) $S^1 \times S^2$ (c) *n*-dimensional torus T^n .

4. Use cellular homology to compute the homology and cohomology of $\mathbb{RP}^2 \times \mathbb{RP}^2$ with integer coefficients. You **do not** need to compute the cup product.