

MAT 215C, Spring 2021
Homework 3

Due before 3:10 on Monday, April 19

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

Let X be a closed n -dimensional manifold, and let \mathring{X} be a complement to a small ball in X .

1. Prove that $H_i(\mathring{X}) = H_i(X)$ for $i \leq n-2$. *Hint: Use Mayer-Vietoris sequence.*
2. If $X = \mathbb{R}P^2$, prove that \mathring{X} is homeomorphic to the Möbius band. Show that in this example $H_1(X) \neq H_1(\mathring{X})$ and the result of problem 1 does not hold for $i = n-1$ in general.
3. Compute the homology of \mathring{X} in terms of homology of X for (a) orientable (b) non-orientable closed surface X .

Recall that if X and Y are n -dimensional manifolds, then their connected sum $X \# Y$ is obtained by gluing \mathring{X} to \mathring{Y} along the common boundary.

3. Show that $\mathbb{R}P^2 \# \mathbb{R}P^2$ is homeomorphic to the Klein bottle.