MAT 215C, Spring 2021
Homework 4
Due before 3:10 on Monday, April 26

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

In all problems in this homework, you can assume that all manifolds are either topological, or smooth, or PL, whatever is easier for you. The results hold in either category.

1. Let $f : M \to N$ be a k-sheeted covering of n-dimensional manifolds. Prove that if N is orientable then M is orientable.

2. Suppose that an n-dimensional manifold is simply connected. Prove that it is orientable.

3. Let N be a non-orientable n-dimensional manifold. Let $\widetilde{N}$ be the set of pairs (a point $x \in N$, local orientation of N at x). Prove that $\widetilde{N}$ is an orientable connected n-dimensional manifold, and there is a 2-sheeted cover $\widetilde{N} \to N$.

4. Let $N = \#^k \mathbb{RP}^2$ be a non-orientable surface, and let $\widetilde{N}$ be its 2-fold orientable cover from Problem 3. Compute the genus of $\widetilde{N}$. *Hint: use Euler characteristics.*