MAT 215C, Spring 2021
Homework 5

Due before 3:10 on Monday, May 3

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. Let M be a compact connected n-dimensional manifold. Prove that there exists a class $w_1(M) \in H^1(M; \mathbb{Z}_2)$ such that for any loop γ in M we have

$$w_1(\gamma) = \begin{cases} 1 & \text{if } \gamma \text{ reverses orientation} \\ 0 & \text{if } \gamma \text{ preserves orientation.} \end{cases}$$

Such w_1 is called the first Stiefel-Whitney class.

2. Consider the change of variables between the Euclidean coordinates (x, y) and polar coordinates (r, φ) on the plane. Does it preserve orientation?

3. Recall that a holomorphic function $f : \mathbb{C} \to \mathbb{C}$, $f(x + iy) = u + iv$ satisfies Cauchy-Riemann equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

Prove that the change of coordinates $(x, y) \to (u, v)$ preserves orientation.

4. Let M be a complex 1-dimensional manifold. This means that M is covered by charts homeomorphic to $\mathbb{C}$ and transition functions are holomorphic. Prove that M is an orientable real 2-dimensional manifold. *Hint: use problem 3.*