

MAT 215C, Spring 2021
Homework 6

Due before 3:10 on Monday, May 10

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

In the problems below $[x_0 : x_1 : x_2]$ denote the homogeneous coordinates on $\mathbb{C}\mathbb{P}^2$.

1. (a) Compute the intersection index of $\mathbb{C}\mathbb{P}^1 \subset \mathbb{C}\mathbb{P}^2$ with itself.
(b) Use part (a) to describe the intersection form on $H_2(\mathbb{C}\mathbb{P}^2)$.
2. Let $f(x_0, x_1, x_2)$ be a homogeneous polynomial of degree d which defines a smooth complex 1-submanifold (and hence smooth real 2-submanifold) $M_f = \{f = 0\}$ of $\mathbb{C}\mathbb{P}^2$.
(a) Compute the intersection index of M_f with a generic line ($\mathbb{C}\mathbb{P}^1$).
(b) Use part (a) to compute the image of the fundamental class $[M_f]$ in $H_2(\mathbb{C}\mathbb{P}^2)$.
3. Let $f(x_0, x_1, x_2)$ and $g(x_0, x_1, x_2)$ be two homogeneous polynomials of degrees p and q . Assume that M_f and M_g are smooth and transversal to each other. In how many points do they intersect?
4. Choose a basis in H_1 of the genus g orientable surface, and compute the intersection form in this basis.