MAT 215C, Spring 2021
Homework 7
Due before 3:10 on Monday, May 17

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. Compute $H_*(M)$ and $H_*(M, \partial M)$ where M is a 2-sphere with k disks removed.

2. Prove that any compact orientable surface is a boundary of some 3-manifold.

3. a) Let M be a compact 3-manifold with boundary (not necessary orientable). Use Poincaré duality with $\mathbb{Z}_2$ coefficients to prove that

$$\chi(M, \partial M) = -\chi(M).$$

b) Prove that $\chi(\partial M) = 2\chi(M)$.

4. Use problem 3 to prove that $\mathbb{RP}^2$ is not a boundary of any compact 3-manifold.