MAT 215C, Spring 2021
Homework 8

Due before 3:10 on Wednesday, May 26

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. Let γ be a simple closed curve (with no self-intersections) on the plane bounding a region of area A. Compute $\int_\gamma xdy$ and $\int_\gamma ydx$ using Stokes theorem.

2. Use de Rham theorem to compute de Rham cohomology of

$$\mathbb{R}^2 \setminus \{(0,0)\}.$$

Find explicit differential forms corresponding to the basis in this cohomology.

3. A 2-form ω on a $2n$-dimensional M is called **symplectic** if $d\omega = 0$ and ω^n does not vanish at any point of M. Prove that ω represents a **nonzero** class in $H^2_{dR}(M)$.

4. Let M and N be two connected oriented smooth n-manifolds, $f : M \to N$ a smooth function and ω an n-form on N. Use de Rham cohomology to prove that

$$\int_M f^*\omega = d \int_N \omega$$

for some integer d. What is the topological meaning of d?