

MAT 215C, Spring 2021  
Homework 9

**Due before 3:10 on Friday, June 4**

*Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.*

1. Prove that the tangent bundle to (a)  $S^1$  (b)  $n$ -dimensional torus is trivial.
2. Describe all surfaces (compact, connected, with no boundary) with trivial tangent bundle.
3. Recall that  $\mathbb{C}\mathbb{P}^1$  is the space of lines in  $\mathbb{C}^2$ , so for every point  $p \in \mathbb{C}\mathbb{P}^1$  there is a corresponding line  $\ell_p \subset \mathbb{C}^2$ . The complex rank 1 bundle  $\mathcal{O}(k)$  is defined as follows: its fiber over  $p$  is given by the space of homogeneous degree  $k$  polynomials on  $\ell_p$ .
  - a) Prove that any homogeneous degree  $k$  polynomial in two variables defines a section of  $\mathcal{O}(k)$ .
  - b) Compute the Euler number of  $\mathcal{O}(k)$ .
4. Similarly to problem 3, one can define the complex rank 1 bundle  $\mathcal{O}(k)$  on  $\mathbb{C}\mathbb{P}^n$ . Compute its Euler class.