

Math 21A, Fall 2021
Practice problems for Midterm 1

This practice sheet contains more problems than the actual exam.

1. Consider the functions:

$$f(x) = \frac{x^2 + 1}{x^2 - 1}, \quad g(x) = ||x + 1| - 1|, \quad h(x) = \frac{1}{\ln x}.$$

For each of them:

- a) Find the domain
 - b) Determine the intersection points of the graph with the coordinate axis
 - c) Find all vertical and horizontal asymptotes (if any)
 - d) Sketch the graph
2. For which c the function

$$f(x) = \begin{cases} \sin x, & \text{if } x < c, \\ \cos x, & \text{if } x \geq c \end{cases}$$

is continuous?

3. Compute the limits:

a)

$$\lim_{x \rightarrow 0} (\sin^3 x - \cos x)$$

b)

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1}}{\sqrt{x}}$$

c)

$$\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 3x}.$$

d)

$$\lim_{x \rightarrow 0} \frac{7x^3 + 8x^2 + 9x}{11x + 12x^2 - 13x^3}.$$

e)

$$\lim_{x \rightarrow +\infty} e^{-x^2}$$

f)

$$\lim_{x \rightarrow +\infty} (\ln(x+1) - \ln x)$$

g)

$$\lim_{x \rightarrow +\infty} \frac{7x^2 + 8x + 9}{11 + 12x - 13x^2}.$$

h)

$$\lim_{x \rightarrow 2^+} \arctan\left(\frac{1}{x-2}\right).$$

i)

$$\lim_{x \rightarrow 0} x^2 \cos(1/x).$$

j)

$$\lim_{x \rightarrow +\infty} \frac{2x^3 + \sqrt{x^4 + 7}}{3x^3 - 5}$$

4. Find $\lim_{x \rightarrow 1} f(x)$, where

$$f(x) = \begin{cases} x^2 + 3x + 4, & \text{if } x \geq 1, \\ 7x^2 + 1, & \text{if } x < 1. \end{cases}$$

5. Plot the graph of the function

$$\operatorname{sgn}(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

Is it continuous?

6. For the function

$$f(x) = \begin{cases} x^2 + x - 9, & \text{if } |x| \geq 3, \\ x, & \text{if } |x| < 3, \end{cases}$$

find one-sided limits at $x = -3$, $x = 0$, $x = 3$. Is this function continuous?

7*. Use the Intermediate Value Theorem to show that the equation

$$x^3 + x - 3 = 0$$

has a solution on the interval $[1, 2]$.