

MAT 21A, Fall 2021
Solutions to HW 1

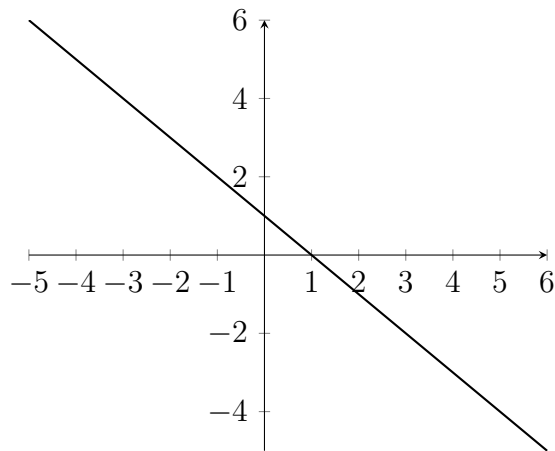
1. (10 points) Graph the function $y = |1 - x| - 1$.

Solution 1: We have $|1 - x| = 1 - x$, if $1 - x \geq 0$ (that is, $x \leq 1$) and $|1 - x| = x - 1$ if $x > 1$. Therefore

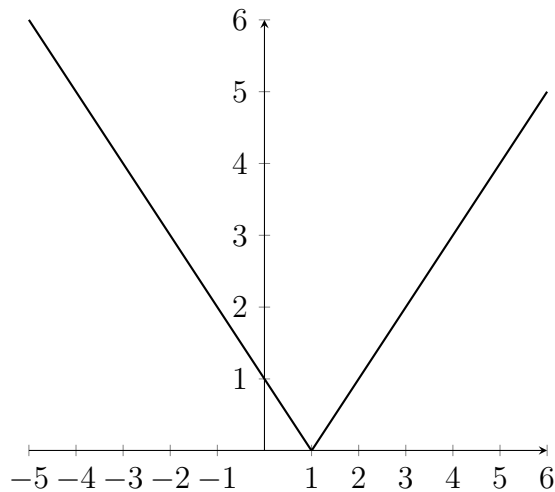
$$y(x) = \begin{cases} -x & \text{if } x \leq 1, \\ x - 2 & \text{if } x > 1. \end{cases}$$

The domain is $(-\infty, +\infty)$. See the graph below.

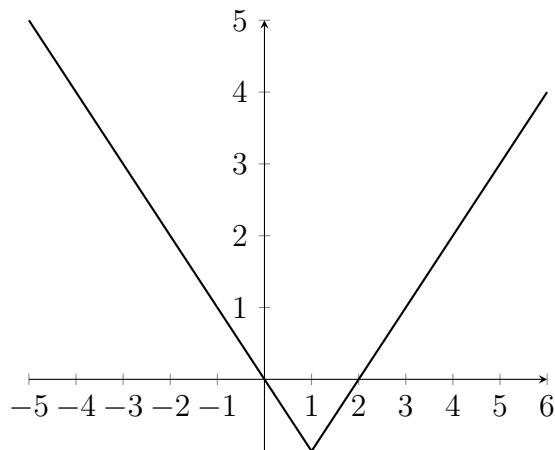
Solution 2: The graph of the linear function $y = 1 - x$ has the form:



The graph of $y = |1 - x|$ is obtained from it by reflection of its negative parts in the horizontal axis:



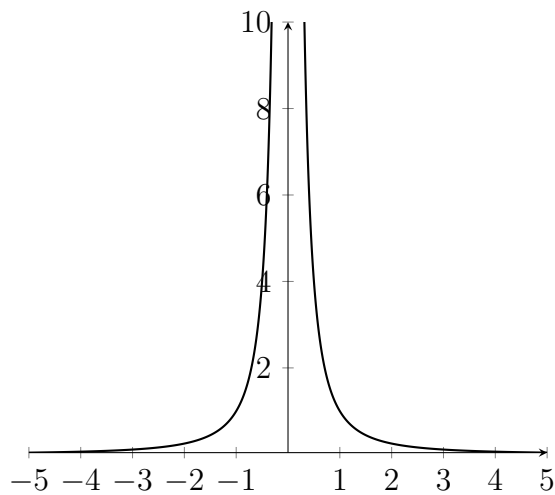
The graph of $y = |1 - x| - 1$ is obtained from it by vertical shift down by 1:



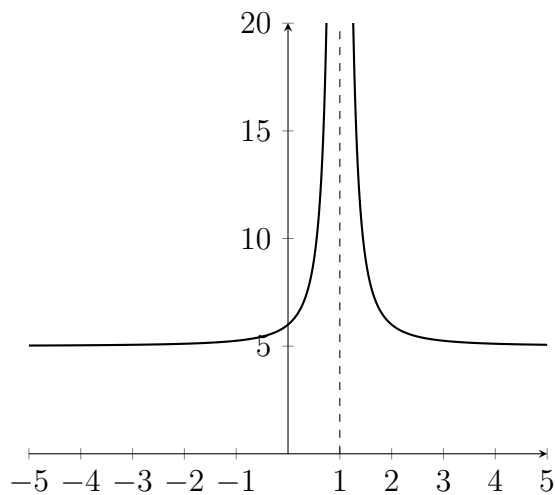
The domain is $(-\infty, +\infty)$.

2. (10 points) Sketch the graph of the function $y = \frac{1}{(x-1)^2} + 5$.

Solution: The graph of $y = \frac{1}{x^2}$ has the form:



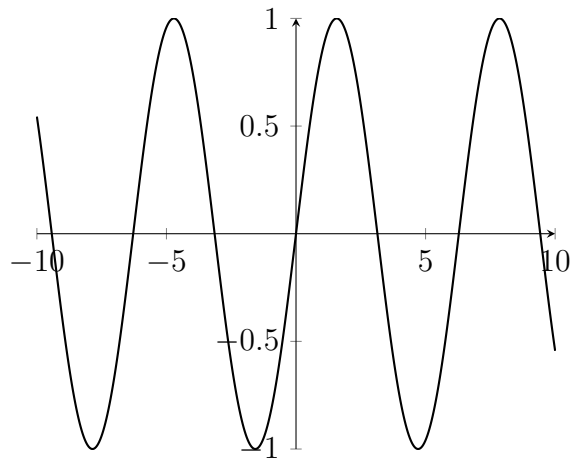
The graph of $y = \frac{1}{(x-1)^2} + 5$ is obtained from it by the shift right by 1 and up by 5:



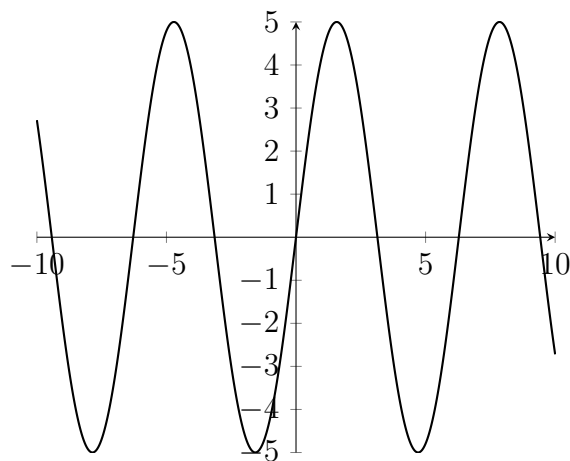
The domain is $(-\infty, 1) \cup (1, +\infty)$.

3. (10 points) Sketch the graph of the function $y = 5 \sin(x) - 1$.

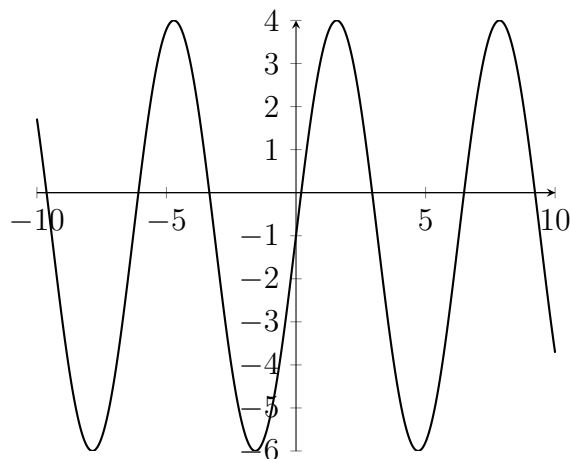
Solution: The graph of $y = \sin(x)$ has the form:



The graph of $y = 5 \sin(x)$ is obtained from it by the vertical stretching by a factor of 5:

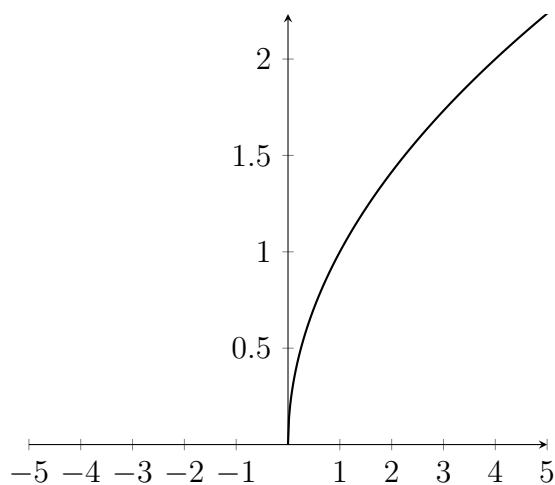


The graph of $y = 5 \sin(x) - 1$ is obtained from it by the vertical shift by (-1) :

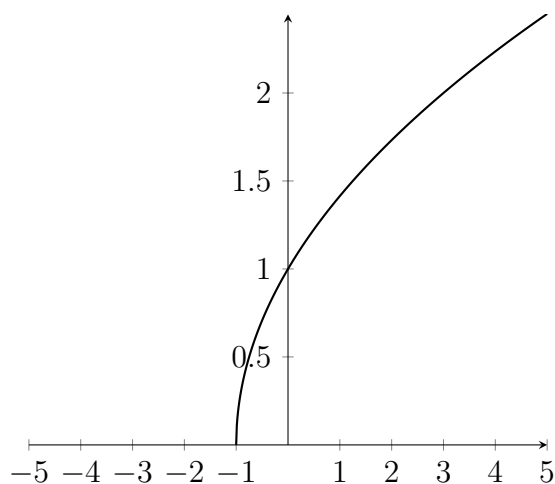


4. (10 points) Sketch the graph of the function $y = \sqrt{1 - \frac{x}{2}}$.

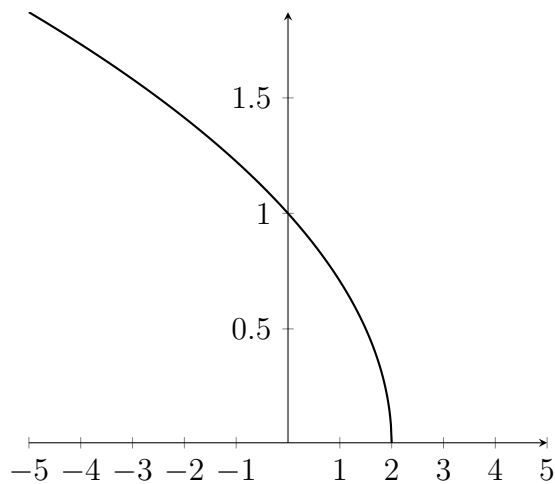
Solution: The graph of $y = \sqrt{x}$ has the form:



The graph of $y = \sqrt{1+x}$ is obtained from it by the horizontal shift left by 1:



Next, we replace x by $-\frac{x}{2}$. This amounts to reflecting the graph in the vertical axis and stretching it horizontally by a factor of 2:



The domain is $(-\infty, 2]$ since the function is defined if $1 - x/2 \geq 0, x \leq 2$.