# MAT 21A, Fall 2021
# Solutions to homework 2

**1.** (10 points) Compute the limit:

$$\lim_{x\to 0} \frac{\frac{1}{x-1}+\frac{1}{x+1}}{x}.$$

**Solution:** Let us simplify the fraction first:

$$\frac{1}{x-1}+\frac{1}{x+1}=\frac{x+1}{(x-1)(x+1)}+\frac{x-1}{(x-1)(x+1)}=\frac{x+1+x-1}{(x-1)(x+1)}=\frac{2x}{(x-1)(x+1)},$$

so

$$\frac{\frac{1}{x-1}+\frac{1}{x+1}}{x}=\frac{2}{(x-1)(x+1)}.$$

Now

$$\lim_{x\to 0} \frac{\frac{1}{x-1}+\frac{1}{x+1}}{x}=\lim_{x\to 0}\frac{2}{(x-1)(x+1)}=\frac{2}{(-1)(1)}=-2.$$

**2.** (10 points) Compute the limit:

$$\lim_{x\to 1}\frac{x^2-x}{x^2-3x+2}.$$

**Solution:** We have

$$\lim_{x\to 1}\frac{x^2-x}{x^2-3x+2}=\lim_{x\to 1}\frac{x(x-1)}{(x-1)(x-2)}=\lim_{x\to 1}\frac{x}{x-2}=\frac{1}{1-2}=-1.$$

**3.** (10 points) If $2-x^2\le g(x)\le 2\cos x$ for all $x$, find $\lim_{x\to 0} g(x)$

**Solution:** We have $\lim_{x\to 0}(2-x^2)=2-0=2$ and $\lim_{x\to 0} 2\cos x=2\cos(0)=2$. Therefore by Squeeze Theorem we have $\lim_{x\to 0} g(x)=2$.

**4.** (10 points) Compute the one-sided limits:

a)

$$\lim_{x\to 1^+}\frac{\sqrt{2x}(x-1)}{|x-1|}.$$

b)

$$\lim_{x\to 1^-}\frac{\sqrt{2x}(x-1)}{|x-1|}.$$

**Solution:** (a) For $x>1$ we have $x-1>0$ and $|x-1|=x-1$. Therefore

$$\lim_{x\to 1^+}\frac{\sqrt{2x}(x-1)}{|x-1|}=\lim_{x\to 1^+}\frac{\sqrt{2x}(x-1)}{x-1}=\lim_{x\to 1^+}\sqrt{2x}=\sqrt{2}.$$

(b) For $x<1$ we have $x-1<0$ and $|x-1|=-(x-1)$. Therefore

$$\lim_{x\to 1^-}\frac{\sqrt{2x}(x-1)}{|x-1|}=\lim_{x\to 1^-}\frac{\sqrt{2x}(x-1)}{-(x-1)}=\lim_{x\to 1^-}-\sqrt{2x}=-\sqrt{2}.$$