MAT 21A, Fall 2021
Solutions to homework 4

1. $f(r) = re^{-r}$

Solution: First we compute the derivative of e^{-r} using Chain Rule:

$$(e^{-r})' = e^{-r}(-r)' = e^{-r}(-1) = -e^{-r}.$$

Next, we use Product Rule:

$$(re^{-r})' = (r)'e^{-r} + r(e^{-r})' = 1 \cdot e^{-r} + r(-e^{-r}) = (1-r)e^{-r}.$$

2. $f(z) = e^z(z-1)(z^2+2)$

Solution: We use the Product Rule twice:

$$(e^z(z-1))' = (e^z)'(z-1) + e^z(z-1)' = e^z(z-1) + e^z = e^z(z-1+1) = e^z z,$$

then

$$f'(z) = (e^z(z-1))'(z^2+2) + e^z(z-1)(z^2-2)' = e^z z(z^2+2) + e^z(z-1)2z =$$
$$e^z(z^3 + 2z + 2z^2 - 2z) = e^z(z^3 + 2z^2).$$

3. $\frac{\sin x}{x} + \frac{x}{\sin x}$.

Solution: By Quotient Rule we have

$$\left(\frac{\sin x}{x} + \frac{x}{\sin x}\right)' = \left(\frac{\sin x}{x}\right)' + \left(\frac{x}{\sin x}\right)' =$$
$$\frac{(\sin x)'x - \sin x(x)'}{x^2} + \frac{(x)'\sin x - x(\sin x)'}{\sin^2 x} =$$
$$\frac{x\cos x - \sin x}{x^2} + \frac{\sin x - x\cos x}{\sin^2 x}.$$

4. Find all points on the graph of $g(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 1$ with tangent lines parallel to the line $y = 4x$.

Solution: The slope of the tangent line equals $g'(x) = \frac{1}{3} \cdot 3x^2 - \frac{3}{2} \cdot 2x = x^2 - 3x$. The tangent line is parallel to $y = 4x$ if its slope is equal to 4, so we get

$$x^2 - 3x = 4,\ x^2 - 3x - 4 = (x-4)(x+1) = 0,$$

so $x = 4$ or $x = -1$.