MAT 21A, Fall 2021
Solutions to homework 5

Find the derivatives of the following functions:

1. $f(x) = \arctan(x^2)$

Solution: We use Chain Rule: $\arctan(x)' = \frac{1}{1+x^2}$, so

$$f'(x) = \frac{1}{1+(x^2)^2} \cdot (x^2)' = \frac{1}{1+x^4} \cdot 2x = \frac{2x}{1+x^4}.$$

2. $f(t) = \left(\frac{3t+4}{5t-2}\right)^{10}$

Solution: We compute the derivtive of the inside function by Quotient Rule:

$$\left(\frac{3t+4}{5t-2}\right)' = \frac{(3t+4)'(5t-2)-(3t+4)(5t-2)'}{(5t-2)^2} = \frac{3(5t-2)-5(3t+4)}{(5t-2)^2} =$$

$$\frac{15t-6-15t-20}{(5t-2)^2} = \frac{-26}{(5t-2)^2}.$$

Now we apply the Chain Rule:

$$f'(t) = 10\left(\frac{3t+4}{5t-2}\right)^9 \left(\frac{3t+4}{5t-2}\right)' = 10\left(\frac{3t+4}{5t-2}\right)^9 \cdot \frac{-26}{(5t-2)^2} =$$

$$\frac{-260(3t+4)^9}{(5t-2)^{11}}.$$

3. $f(x) = \ln\left(\frac{x+1}{x-1}\right)$

Solution 1: We compute the derivtive of the inside function by Quotient Rule:

$$\left(\frac{x+1}{x-1}\right)' = \frac{(x+1)'(x-1)-(x+1)(x-1)'}{(x-1)^2} = \frac{(x-1)-(x+1)}{(x-1)^2} =$$

$$\frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x-1)^2}.$$

Now we apply Chain Rule:

$$f'(x) = \frac{1}{\left(\frac{x+1}{x-1}\right)} \cdot \left(\frac{x+1}{x-1}\right)' = \frac{x-1}{x+1} \cdot \frac{-2}{(x-1)^2} = \frac{-2}{(x-1)(x+1)}.$$

Solution 2: We can rewrite $\ln\left(\frac{x+1}{x-1}\right) = \ln(x+1) - \ln(x-1)$, so by Chain Rule

$$f'(x) = \frac{1}{x+1} \cdot 1 - \frac{1}{x-1} \cdot 1 = \frac{1}{x+1} - \frac{1}{x-1} = \frac{(x-1)-(x+1)}{(x-1)(x+1)} = \frac{-2}{(x-1)(x+1)}.$$

4. A curve (called folium of Descartes) is defined by the equation

$$x^3 + y^3 - 9xy = 0$$

Find the slopes of tangent lines to this curve at points $(4, 2)$ and $(2, 4)$.

Solution: We use implicit differentiation to find the derivative y'. If we take the derivatives of both sides of the equation of the curve we get

$$3x^2 + 3y^2y' - 9y - 9xy' = 0, \ (3x^2 - 9y) + y'(3y^2 - 9x) = 0,$$

so

$$y'(3y^2 - 9x) = -(3x^2 - 9y), \ y' = -\frac{(3x^2 - 9y)}{(3y^2 - 9x)}.$$

At $(x, y) = (4, 2)$ we get

$$y' = -\frac{3 \cdot 16 - 9 \cdot 2}{3 \cdot 4 - 9 \cdot 4} = -\frac{48 - 18}{12 - 36} = -\frac{30}{-24} = \frac{5}{4}.$$

Similarly, at $(x, y) = (2, 4)$ we get

$$y' = -\frac{3 \cdot 4 - 9 \cdot 4}{3 \cdot 16 - 9 \cdot 2} = -\frac{-24}{30} = \frac{4}{5}.$$