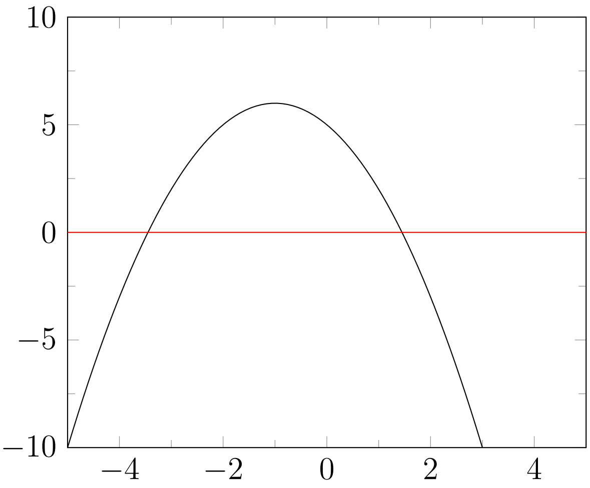
MAT 21A, Fall 2021
Solutions to homework 6

In the first three problems:

- Find the intervals where the function is increasing or decreasing
- Find all local maximums and minimums
- Graph the function using this information

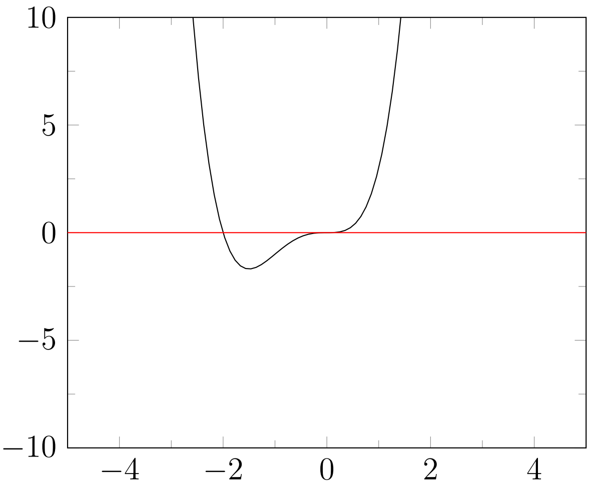
1. $f(x) = 5 - 2x - x^2$

Solution: We have $f'(x) = -2 - 2x$, so $f'(x) > 0$ for $-2 - 2x > 0, 2x < -2, x < -1$. The function is increasing on $(-\infty, -1]$ and decreasing on $[-1, +\infty)$, and has a local maximum at $x = -1$.



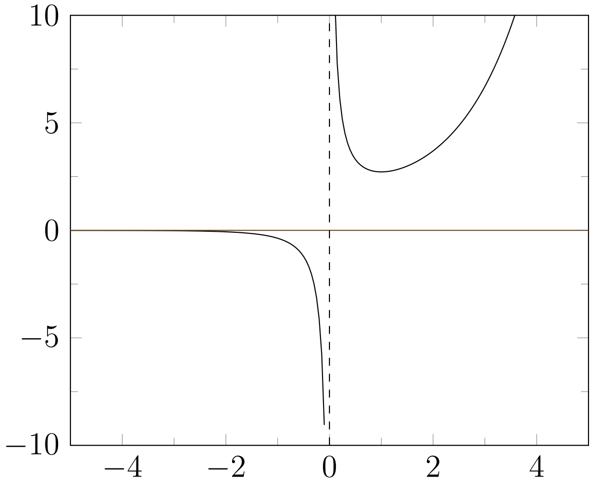
2. $f(x) = x^4 + 2x^3$

Solution: We have $f'(x) = 4x^3 + 6x^2$, so $f'(x) > 0$ for $4x^3 + 6x^2 > 0$. We can factor it as $(4x + 6)x^2$ and $x^2 \geq 0$, so $f'(x) > 0$ if $4x + 6 > 0, 4x > -6, x > -\frac{6}{4} = -\frac{3}{2}$. The function is increasing on $[-\frac{3}{2}, +\infty]$ and decreasing on $(-\infty, -\frac{3}{2}]$, and has a local minimum at $x = -\frac{3}{2}$.



3. $f(x) = \frac{e^x}{x}$

Solution: We have $f'(x) = \frac{e^x \cdot x - e^x \cdot 1}{x^2} = \frac{e^x(x-1)}{x^2}$. Since $e^x > 0$ and $x^2 \geq 0$, we have $f'(x) > 0$ if $x - 1 > 0$, so $x > 1$. The function is increasing on $[1, +\infty]$ and decreasing on $(-\infty, 0)$ and $(0, 1]$ (note that the function is defined for $x \neq 0$), and has a local minimum at $x = 1$.



To sketch the graph, we need to find the asymptotes. At $x = 0$ we have $\lim_{x\to 0} \frac{e^x}{x} = \infty$, so there is a vertical asymptote at $x = 0$. At $x \to -\infty$ we have $e^x \to 0, x \to \infty, so \frac{e^x}{x} \to 0$. At $x \to +\infty$ we have $e^x \gg x$, so $\frac{e^x}{x} \to \infty$. Therefore there is a horizontal asymptote $y = 0$ at $x \to -\infty$.

4. Find the absolute maximum and the absolute minimum of the function$f(x) = e^{-3x^2}$ on the interval $[-2, 1]$.

Solution: By Chain Rule we have $f'(x) = e^{-3x^2}(-3x^2)' = e^{-3x^2} \cdot (-6x)$. The critical point is at $x = 0$, so we need to compare

$$f(-2) = e^{-12},\ f(0) = e^0 = 1,\ f(1) = e^{-3}.$$

The maximal value is 1 at $x = 0$ and the minimal value is e^{-12} at $x = -2$.