In the first three problems:

- Find the intervals where the function is increasing or decreasing
- Find all local maximums and minimums
- Graph the function using this information

1. \( f(x) = 5 - 2x - x^2 \)

   **Solution:** We have \( f'(x) = -2 - 2x \), so \( f'(x) > 0 \) for \(-2 - 2x > 0, 2x < -2, x < -1\). The function is increasing on \(( -\infty, -1]\) and decreasing on \([-1, +\infty)\), and has a local maximum at \( x = -1 \).

2. \( f(x) = x^4 + 2x^3 \)

   **Solution:** We have \( f'(x) = 4x^3 + 6x^2 \), so \( f'(x) > 0 \) for \( 4x^3 + 6x^2 > 0 \). We can factor it as \((4x + 6)x^2\) and \( x^2 \geq 0 \), so \( f'(x) > 0 \) if \( 4x + 6 > 0 \), \( 4x > -6 \), \( x > -\frac{6}{4} = -\frac{3}{2} \). The function is increasing on \([-\frac{3}{2}, +\infty]\) and decreasing on \(( -\infty, -\frac{3}{2}]\), and has a local minimum at \( x = -\frac{3}{2} \).

3. \( f(x) = \frac{e^x}{x} \)

   **Solution:** We have \( f'(x) = \frac{e^x \cdot x - e^x \cdot 1}{x^2} = \frac{e^x(x-1)}{x^2} \). Since \( e^x > 0 \) and \( x^2 \geq 0 \), we have \( f'(x) > 0 \) if \( x - 1 > 0 \), so \( x > 1 \). The function is increasing on \([1, +\infty]\) and decreasing on \(( -\infty, 0)\) and \((0, 1] \) (note that the function is defined for \( x \neq 0 \)), and has a local minimum at \( x = 1 \).
To sketch the graph, we need to find the asymptotes. At $x = 0$ we have $\lim_{x \to 0} \frac{e^x}{x} = \infty$, so there is a vertical asymptote at $x = 0$. At $x \to -\infty$ we have $e^x \to 0$, $x \to \infty$, so $\frac{e^x}{x} \to 0$. At $x \to +\infty$ we have $e^x \gg x$, so $\frac{e^x}{x} \to \infty$. Therefore there is a horizontal asymptote $y = 0$ at $x \to -\infty$.

4. Find the absolute maximum and the absolute minimum of the function $f(x) = e^{-3x^2}$ on the interval $[-2, 1]$.

**Solution:** By Chain Rule we have $f'(x) = e^{-3x^2}(-3x^2)' = e^{-3x^2} \cdot (-6x)$. The critical point is at $x = 0$, so we need to compare

$$f(-2) = e^{-12}, \quad f(0) = e^0 = 1, \quad f(1) = e^{-3}.$$

The maximal value is 1 at $x = 0$ and the minimal value is $e^{-12}$ at $x = -2$. 