MATH 21B, practice problems for Midterm 2

This practice sheet contains more problems than the actual exam.

1. Consider the function $f(x) = \frac{1}{2}(e^x + e^{-x})$.

a) Find the length of the curve given by the equation $y = f(x)$, $-1 \le x \le 1$.

b) Let R be the region bounded by the graph of $f(x)$ and the lines $x = 1, x = -1$ and $y = 0$. Find the area of R.

c) Find the coordinates of the center of mass of R.

d) Consider the solid obtained by rotation of R about the x-axis. Find its volume and surface area.

e) Consider the solid obtained by rotation of R about the y-axis. Find its volume.

2. A plate is bounded by the parabola $y = x^2$ and the line $y = 1$. Find its center of mass.

3. Compute the following integrals:

a)
$$\int \frac{3x + 5}{x^2 + 2x} dx$$

b)
$$\int \frac{3x + 5}{x^2 + 2x + 2} dx$$

c)
$$\int \sin(3x) \cos(5x) dx$$

d)
$$\int x\sqrt{4 - x^2} dx$$

e)
$$\int x^2 \ln x \, dx$$

4. Compute the following definite integrals:

a)
$$\int_2^3 \frac{3x + 5}{x^2 + 2x + 1} dx$$

b)
$$\int_0^\pi x \sin(5x) dx$$

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c)

$$\int_0^1 \frac{x^3 + 2x^2 + 3x + 4}{x + 1} dx$$

d)

$$\int_0^1 \frac{x^3 + 2x^2 + 3x + 4}{x^2 + 1} dx$$

e)

$$\int_0^\pi \sin^3 x \cos^5 x \, dx$$