

MATH 21B, practice problems for the final exam

This practice sheet contains more problems than the actual exam.

1. Consider the region R bounded by the graph of $y = \sin(x)$ on $[0, \pi]$ and the x -axis.
 - a) Find the area of R
 - b) Find the volume of the solid of revolution obtained by rotation of R about the x -axis
 - c) Find the volume of the solid of revolution obtained by rotation of R about the y -axis
 - d) Find the coordinates of the center of mass of R .
2. Compute the following integrals:
 - a) $\int \frac{3x+5}{x^2+2x} dx$
 - b) $\int \frac{3x+5}{x^2+2x+2} dx$
 - c) $\int \sin(3x) \cos(5x) dx$
 - d) $\int x\sqrt{4-x^2} dx$
 - e) $\int x \ln x dx$
3. Compute the following definite integrals:
 - a) $\int_2^3 \frac{3x+5}{x^2+2x+1} dx$
 - b) $\int_0^\pi \sin^3 x \cos^5 x dx$
 - c) $\int_2^3 \frac{dx}{x \ln^2 x}$
 - d) $\int_0^1 \frac{x dx}{\sqrt{4-x^2}}$
 - e) $\int_0^\pi x \sin x dx$.
4. For the following indefinite integrals, determine if they converge or diverge. If they converge, find their values.
 - a) $\int_0^1 \frac{dx}{\sqrt{x}}$
 - b) $\int_1^\infty \frac{dx}{\sqrt{x}}$
 - c) $\int_0^\infty x^2 e^{-x} dx$
 - d) $\int_0^\infty \frac{dx}{x^2+3x+2}$
 - e) $\int_0^\infty \frac{x dx}{x^2+3x+2}$
5. Find the length of the curve $y = \frac{1}{2}x^2$, $1 \leq x \leq 2$.
6. Determine the surface area of the solid obtained by rotating $y = x^3$, $1 \leq x \leq 2$, about the x -axis.
7. A curve is given in polar coordinates by the equation $R = \cos(3\theta)$.
 - a) Find $\frac{dy}{dx}$ as a function of θ .
 - b) Find the equations of the tangent lines to the curve at $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{2}$.

8. A curve is given by a parametric equation

$$x(t) = 4 \cos(t) + \cos(4t), \quad y(t) = 4 \sin(t) - \sin(4t), \quad 0 \leq t \leq \frac{2\pi}{5}.$$

- a) Find $\frac{dy}{dx}$ as a function of θ .
- b) Find the length of the curve.