lecture 17 | **Ex 1** $X = \mathbb{P}^1$ $E = \mathcal{O}(1)$ $\Gamma(\mathcal{O}(1))$ = linear poly

section $s_i = \alpha_i x_0 + \beta_i x_1$

$$f([x_0 : x_1]) = [\alpha_0 x_0 + \beta_0 x_1 : \cdots : \alpha_n x_0 + \beta_n x_1]$$

$$= x_0 [\alpha_0 : \cdots : \alpha_n] + x_1 [\beta_0 : \cdots : \beta_n]$$

line $f : \mathbb{P}^1 \to \mathbb{P}^n$

Ex 2 $X = \mathbb{P}^1$, $E = \mathcal{O}(3)$

$$[s_0 : s_1 : s_2 : s_3] = [x_0^3 : x_0^2 x_1 : x_0 x_1^2 : x_1^3]$$

twisted cubic

Ex 3 More generally $X = \mathbb{P}^1$, $E = \mathcal{O}(k)$

$$[s_0 : \cdots : s_k] = [x_0^k : x_0^{k-1} x_1 : \cdots : x_1^k]$$

$f : \mathbb{P}^1 \to \mathbb{P}^k$ "Veronese curve"

Rmk In Examples 2 and 3 we pick s_i = basis in $\Gamma(E)$. This is often a good choice, and gives a map

$$f : X \dashrightarrow \mathbb{P}(\Gamma(X, E)) = \mathbb{P}^{m-1} \qquad m = \dim \Gamma(X, E)$$

This is regular if s_i do not vanish simultaneously at some point p. In this case, all sections of E vanish at p (p = "basepoint").

Thm $X \subset \mathbb{P}^n$ <u>irreducible</u> projective alg. set

Then any global funct $X \to \mathbb{K}$ is constant

Proof: Assume $\varphi : X \to \mathbb{K}$ is regular.

$U_i = \{x_i \neq 0\}$ $\quad X \cap U_i =$ alg. set in $U_i = \mathbb{A}^n$

$$\varphi|_{X \cap U_i} = g\left(\frac{x_0}{x_i}, \ldots \frac{x_n}{x_i}\right) = \frac{h(x_0 \ldots x_n)}{x_i^{d_i}} \leftarrow \text{homog deg} = d_i$$

$x_i^{d_i}\varphi$ = polynomial of degree d_i

Pick $N >> \sum d_i$ $\quad G = x_0^{N_0} \ldots x_n^{N_n}$ with $\sum N_i = N >> \sum d_i$

At least for some i $N_i > d_i \Rightarrow x_i^{N_i}\varphi$ = polynomial of deg N_i

$\Rightarrow G\varphi$ is a polynom. of deg N

So $G\varphi$ is a polynomial for any monomial G of degree ~~N~~ of deg N.

$\Rightarrow G \cdot \varphi^k$ is a polynomial of degree N

($G\varphi$ = poly of deg $N \Rightarrow G\varphi^2$ is a poly of deg N etc).

$$\frac{\mathbb{K}[x_0 \ldots x_n]}{I(X)} \subset \frac{\mathbb{K}[x_0 \ldots x_n]}{I(X)} \oplus \varphi \frac{\mathbb{K}[x_0 \ldots x_n]}{I(X)} \subset \ldots \subset G^{-1}\frac{\mathbb{K}[x_0 \ldots x_n]}{I(X)}$$

Since $\frac{\mathbb{K}[x]}{I(X)}$ is Noetherian, this stabilizes.

$$\varphi^k = a_{k-1}\varphi^{k-1} + \ldots + a_0 \quad a_i \in \mathbb{K}[x_0 \ldots x_n]$$

mod $I(X)$

φ has degree $0 \Rightarrow$ $\varphi^k = a_{k-1}(0)\varphi^{k-1} + \ldots + a_0(0)$

$$\prod_i (\varphi - r_i) = 0 \mod I(X)$$

$\Rightarrow X = \bigcup_i \{\varphi = r_i\}$. Since X is irred, $\varphi = r_i$ for some i.