

MAT 248A, Winter 2026

Homework 5

Due before 3:00pm on Friday, February 13

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. Describe the preimages of the curves under the blowup of \mathbb{C}^2 at the origin:
 - a) $\{xy = 0\}$
 - b) $\{x^2 = y^3\}$
 - c) $\{x^3 = y^5\}$
2. Prove that the set of $m \times n$ matrices of maximal rank is a Zariski open subset of the space \mathbb{A}^{mn} of all matrices. *Hint: a matrix has maximal rank if at least one of its maximal minors is nonzero*
3. Let $M_k(m \times n)$ be the set of $m \times n$ matrices of rank exactly k .
 - a) Describe the sets $M_k(2 \times 2)$ for all k by explicit equations (and inequalities).
 - b) Given $1 \leq i_1 < \dots < i_k \leq m$, let U_{i_1, \dots, i_k} be the subset of $M_k(m \times n)$ such that the rows i_1, \dots, i_k are linearly independent. Prove that it is Zariski open in $M_k(m \times n)$.
 - c) Prove that U_{i_1, \dots, i_k} is isomorphic to a Zariski open subset in some affine space \mathbb{A}^N and compute N .
4. Let $X \subset \mathbb{A}^4 \times \mathbb{P}^1$ be the space of pairs (A, ℓ) where $A \in \mathbb{A}^4$ is a 2×2 matrix, $\ell \in \mathbb{P}^1$ is a line in \mathbb{A}^2 through the origin, and $A\ell = 0$.
 - a) Prove that X is a Zariski closed subset of $\mathbb{A}^4 \times \mathbb{P}^1$.
 - b) Describe the fibers of the projection $X \rightarrow \mathbb{A}^4$.
 - c) Describe the fibers of the projection $X \rightarrow \mathbb{P}^1$.