MAT 261A, Spring 2023
Homework 1
Due before 12:10 on Monday, April 10

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. The group $GL(n, \mathbb{R})$ acts on $\mathbb{R}^n$ by $A(v) = Av$.
  a) Find the orbit of the basis vector $e_1 = (1, 0, \dots, 0)$ under this action.
  b) Find the stabilizer of e_1 under this action.

2. The orthogonal group $O(n, \mathbb{R})$ acts on $\mathbb{R}^n$ by $A(v) = Av$.
  a) Find the orbit of e_1 under this action.
  b) Find the stabilizer of e_1 under this action.

3. A matrix X is called skew-symmetric if $X + X^T = 0$.
  a) Prove that skew-symmetric matrices form a vector space and find the dimension of this space.
  b) Let A be an orthogonal matrix and X a skew-symmetric one. Prove that $A^T X A$ is a skew-symmetric matrix.
  c) Prove that (b) defines an action of the orthogonal group $O(n)$ on the space of skew-symmetric matrices.

4. Prove that the following are matrix Lie groups:
  a) All invertible diagonal matrices.
  b) All invertible upper-triangular matrices.
  c) All upper-triangular matrices with 1 on diagonal.