

MAT 261A, Spring 2023  
Homework 4

**Due before 12:10 on Monday, May 1**

*Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.*

1. The Lie algebra  $\mathfrak{sl}_2$  consists of traceless  $2 \times 2$  matrices (over  $\mathbb{R}$  or over  $\mathbb{C}$ ) and has basis

$$H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad F = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Compute the commutators  $[H, E]$ ,  $[H, F]$  and  $[E, F]$ .

2. The Lie algebra  $\mathfrak{so}_3$  consists of real skew-symmetric  $3 \times 3$  matrices. Find the basis in  $\mathfrak{so}_3$  and compute all commutators in this basis.

3. The Lie algebra  $\mathfrak{su}_2$  consists of complex  $2 \times 2$  matrices  $X$  such that  $X^* = -X$  and  $\text{Tr}(X) = 0$ . Find the basis in  $\mathfrak{su}_2$  over  $\mathbb{R}$  and compute all commutators in this basis.

4. Use problems 1-3 to prove the following:

- a) The Lie algebras  $\mathfrak{so}_3$  and  $\mathfrak{su}_2$  are isomorphic.
- b) The **complexified** Lie algebras  $\mathfrak{so}_3 \otimes \mathbb{C}$ ,  $\mathfrak{su}_2 \otimes \mathbb{C}$  and

$$\mathfrak{sl}_2(\mathbb{R}) \otimes \mathbb{C} = \mathfrak{sl}_2(\mathbb{C})$$

are all isomorphic.