MAT 261A, Spring 2023
Homework 6
Due before 1:10 on Monday, May 15

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

All problems are about representations of the Lie algebra $\mathfrak{sl}_2$.

1. Use characters to prove $L(1) \otimes L(n) \simeq L(n+1) \oplus L(n-1)$.

2. Decompose $L(a) \otimes L(b)$ into irreducible representations for all a, b.

3. Decompose $L(1) \otimes L(1) \otimes L(1)$ into irreducible representations.

4. Prove that for all n the tensor product $\underbrace{L(1) \otimes \cdots \otimes L(1)}_{n}$ contains a unique copy of $L(n)$ and all other irreducible summands are of the form $L(j), j < n$.

5*.[1] Find the multiplicity of the trivial representation $L(0)$ in $\underbrace{L(1) \otimes \cdots \otimes L(1)}_{n}$ for all n.

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[1]This is a bonus problem