Characteristic classes, Fall 2023
Homework 2

1. Compute $\omega_i(TM)$ where M is a genus g surface.

2. Suppose that an n-dimensional manifold M can be immersed in $\mathbb{R}^{n+1}$. Show that $\omega_i(TM) = (\omega_1(TM))^i$.

3. Consider the standard embedding of $\mathbb{RP}^k$ into $\mathbb{RP}^n$ for $k < n$. Find all Stiefel-Whitney classes of the normal bundle for this embedding.

4. Prove the following theorem of Stiefel. If $n+1 = 2^r m$ with m odd, then there do not exist 2^r vector fields on the projective space $\mathbb{RP}^n$ which are everywhere linearly independent.

5. A manifold M is said to admit a field of tangent k-planes if its tangent bundle admits a sub-bundle of dimension k.
   a) Show that $\mathbb{RP}^n$ admits a field of tangent 1-planes if and only if n is odd.
   b) Show that $\mathbb{RP}^4$ and $\mathbb{RP}^6$ do not admit fields of tangent 2-planes.