Research activities (2009 - present)

For the last one and half years, my research has been focused on developing compressive imaging methods.

Compressive imaging is the idea of imaging *sparse* objects with comparably *sparse* measurements. The approach is based the compressed sensing (CS) theory originating in information theory and statistics. In the standard theory, problems of imaging are typically formulated in the continuum setting and results are usually qualitative. For example, the standard inverse scattering theory asserts the uniqueness of scatterer given the entirety of scattering data (i.e. all incident and scattering angles). But in reality one has only a *finite* (sometimes small) number of scattering data, let alone the complete continuum set of scattering data!

From this perspective, CS is a natural, alternative route to inverse scattering and, more generally, imaging. As such, compressive imaging has seen a flurry of activities in the last few years. Few, though, really address the heart of the problem. A simple fact is that the striking results of CS demand equally striking assumptions on sensing matrices and objects of interest. Moreover, for problems of imaging physical constraints often result in sensing matrices not previously analyzed from the CS perspective. How does one conduct measurements and collect data in such a way that these desirable properties are *provably* met under all the constraints of wave physics? This question is typically evaded in most literature on this topic which is exactly the main focus of my research.

For example, in a series of papers I reformulate the far-field and near-field imaging problems in the CS framework and put compressive imaging on a rigorous ground by proposing novel measurement schemes and establishing various desirable CS properties for the resulting measurement matrices depending on the imaging geometry and the physical constraints. The paper [1] dealing with the single-input-multiple-output (SIMO) measurement schemes has been on the list of 10 most read papers of the journal *Inverse Problems* since its publication in February 2010.

Major findings

Here is the brief summary of my research findings reported in 5 papers listed in the end of the summary.

- [1] This paper puts forth for the first time a rigorous compressed sensing analysis of inverse scattering with point scatterers and the single-input-multiple-output (SIMO) measurements.

  The proposed single- or multiple-shot SIMO measurements are general and give rise to a type of sensing matrices not analyzed previously which require *high frequency* waves in order to have a performance guarantee.

  By duality theory for scattering, various equivalent multiple-input-single-output (MISO) and multiple-input-multiple-output (MIMO) schemes are also presented and analyzed.

  In the absence of noise, it is shown that the ℓ^1-minimization principle, called Basis Pursuit (BP), can recover exactly the target of sparsity on the order of the number of scattering data. The stability with respect to noisy data is proved for weak or widely separated scatterers under a stronger sparsity constraint.

- [2] This paper proposes and analyzes the multiple-shot, single-input-single-output (SISO) measurements. For this type of measurement schemes the probe frequencies, the incident and the sampling directions are related in a precise way. Both point and extended targets are considered.

  Two schemes are particularly interesting: The first one employs multiple frequencies with the sampling angle always in the back-scattering direction resembling the synthetic aperture (SA) imaging; the second employs only single frequency with the sampling angle in the (nearly) forward scattering direction in the high frequency limit, resembling the setting of X-ray tomography.

  One striking finding is that for distributed extended targets represented in the Littlewood-Paley basis a specially designed sampling scheme then transforms the scattering matrix into a *block-diagonal* matrix with each block being the random Fourier matrix corresponding to one of the multiple dyadic scales of the extended target. In other words the different dyadic scales of the target are decoupled and therefore can be reconstructed scale-by-scale by the proposed method. Moreover, with probes of any single frequency the coefficients in the Littlewood-Paley expansion for scales up to wavelength can be exactly recovered. Hence the classical resolution criterion in a different form is proven valid for this imaging set-up.
- [3] The problem of imaging *extended* targets (sources or scatterers) is formulated in the framework of compressed sensing with emphasis on *subwavelength* resolution using *near field*.

  We show that the number of the stably resolvable modes typically increases as the negative d-th (the dimension of the target) power of the distance between the target and the sensors/source. The resolution limit is shown to be inversely proportional to the SNR in the high SNR limit. In other words, the classical resolution limit can be overcome with near field CS measurement schemes.
- [4] The main finding of this paper is the discovery of *threshold* aperture for which the proposed far-field measurement scheme has a CS performance guarantee.

  This notion of threshold aperture is equivalent to the classical Rayleigh criterion in traditional imaging and implies that the proposed measurement scheme does not sacrifice the standard resolution for the gain in sparsity.

  Our numerical results indicate that the threshold aperture is a genuine limit below which the CS techniques have a poor performance.
- [5] The MUSIC algorithm, and its extension for imaging sparse *extended* objects, with noisy data is analyzed by compressed sensing (CS) techniques. The MUSIC algorithm was invented in mid 1980's and became popular due to its simplicity, computational efficiency and robustness with respect to noise. But to my best knowledge its performance has never been rigorously analyzed, especially in the case of noisy data. The existing performance guarantee for the case of noiseless data is only qualitative in nature.

  Making a surprising link with CS through the notion of restricted isometry constant (RIC) I establish sufficient, quantitative conditions for the exact localization by MUSIC with or without noise. In the noiseless case, the sufficient condition gives an upper bound on the numbers of random sampling and incident directions necessary for exact localization. In the noisy case, the sufficient condition assumes additionally an upper bound for the noise-to-object ratio in terms of the RIC and the dynamic range of objects.

Rigorous comparison of performance between MUSIC and the CS minimization principle, Basis Pursuit Denoising (BPDN), is given. In general, the MUSIC algorithm guarantees to recover, with high probability, s scatterers with $n = \mathcal{O}(s^2)$ random sampling and incident directions and sufficiently high frequency.

For the favorable imaging geometry where the scatterers are distributed on a transverse plane MUSIC guarantees to recover, with high probability, s scatterers with a median frequency and $n = \mathcal{O}(s)$ random sampling/incident directions.

Moreover, for the problems of spectral estimation and source localizations both BPDN and MUSIC guarantee, with high probability, to identify exactly the frequencies of random signals with the number $n = \mathcal{O}(s)$ of sampling times. However, in the absence of abundant realizations of signals, BPDN is the preferred method for spectral estimation. Indeed, BPDN can identify the frequencies approximately with just *one* realization of signals with the recovery error at worst linearly proportional to the noise level.

Numerical results confirm that BPDN outperforms MUSIC in the well-resolved case while the opposite is true for the under-resolved case. The latter effect indicates the superresolution capability of the MUSIC algorithm.

References

[1] A. C. Fannjiang, "Compressive inverse scattering I. High-frequency SIMO/MISO and MIMO measurements," *Inverse Problems* **26** (2010), 035008.

[2] A.C. Fannjiang, "Compressive inverse scattering II. multi-shot SISO measurements with Born scatterers," *Inverse Problems* **26** (2010), 035009

[3] A. C. Fannjiang, "Compressive imaging of subwavelength structures," *SIAM J. Imag. Sci.* **2** (2009), 1277-1291.

[4] A. C. Fannjiang, P. Yan and T. Strohmer, "Compressed remote sensing of sparse object," *SIAM J. Imag. Sci.* (in press).

[5] A. C. Fannjiang, " The MUSIC algorithm for sparse objects: a compressed sensing analysis," `arXiv:1006.1678`.