General second-order autonomous equations

In general, a second-order autonomous equation has

- one independent variable and
- one dependent variable.

It has the form $\frac{d^2y}{dt^2} = f\left(y, \frac{dy}{dt}\right)$.

Example. Simple mass-spring system

Hooke's Law: The restoring force of the spring is proportional to the displacement from its rest position.

Using Newton's law $F = ma$, we get

Let's consider the special case where $k = m$. We get $\frac{d^2y}{dt^2} = -y$, and we can guess some solutions to this equation:

General 2D first-order autonomous systems

In general, a 2D first-order autonomous system of ordinary differential equations has

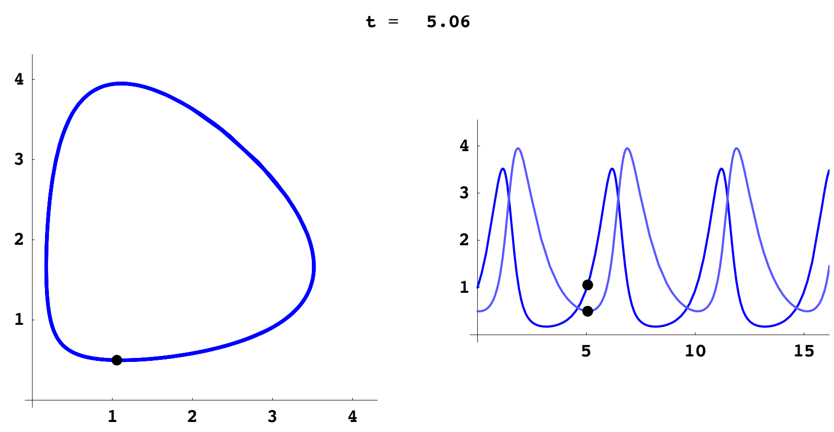
- one independent variable and
- two dependent variables.
- The independent variable does not appear on the right-hand sides of the differential equations.

Example. Recall the predator-prey systems

$$\frac{dR}{dt} = aR - bRF$$
$$\frac{dF}{dt} = -cF + dRF.$$

Let's go through some terminology:

- initial condition:
- solution to an initial-value problem:



The solution shown above corresponds to the initial condition $(R_0, F_0) = (1, 0.5)$ with parameter values $a = 2$, $b = 1.2$, $c = 1$, and $d = 0.9$. See the web site for the entire animation and for a related 3D animation. `DETools` also has a tool called `PredatorPrey`.

- component graphs:

- phase plane:

- solution curve in the phase plane:

- equilibrium solutions:

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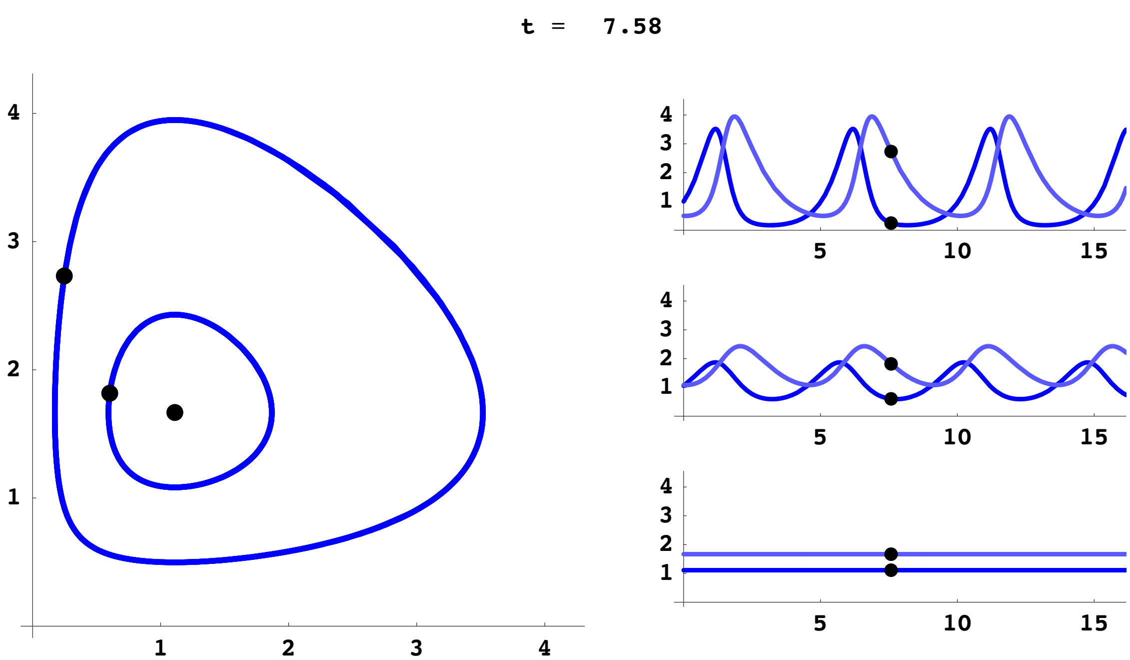
• phase portrait:



One skill that you will learn is how to make a rough sketch of the component graphs from the solution curve. There is a tool on your CD called `DESketchPad` which will help you practice.