A little more on the steady-state solution

I still owe you an explanation for why I prefer to calculate the steady-state solution using complex numbers.

On Friday, we calculated the steady-state solution

$$y_p(t) = -\tfrac{1}{4}(\cos 2t - \sin 2t)$$

for the equation

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 2y = \cos 2t,$$

and we did so using computations that involved complex numbers. In fact, we found $y_p(t)$ as the real part of

$$y_c(t) = -\tfrac{1}{4}(1+i)\, e^{(2i)t}.$$

The complex number

$$a = -\tfrac{1}{4}(1+i)$$

tells us everything we need to know about the steady-state solution.

In order to see why, we use polar coordinates in the complex plane (see pp. 745–747 in Appendix C of the text).

Let's rewrite $a = -\frac{1}{4}(1+i)$ in this polar form.

What does this polar representation of a tell us about the steady-state solution?

Sinusoidal forcing in the absense of damping

Now consider the mass-spring system without the dashpot.

Example. Let's find the general solution to

$$\frac{d^2y}{dt^2} + 3y = \cos \omega t.$$

Note the lack of a damping term. We want to see what happens with various forcing frequencies.

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Unfortunately the parts of the solution that correspond to the associated homogeneous equation do not die out. So to get some qualitative understanding in this case, we make a simplifying assumption. We consider the solution that satisfies the initial condition $(y(0), y'(0)) = (0, 0)$.

On the web site, there is a Quicktime animation of the graphs of these solutions as we vary the forcing frequency ω. We can also visualize these solutions using a parameter in `HPGSystemSolver`.