
Autonomous Differential Equations

A first-order differential equation with independent variable t and dependent variable y is **autonomous** if

$$\frac{dy}{dt} = f(y).$$

The rate of change of $y(t)$ depends only on the value of y .

Examples of autonomous equations: exponential growth model, radioactive decay, logistic population model

Example. $\frac{dv}{dt} = -kv + a \sin bt$

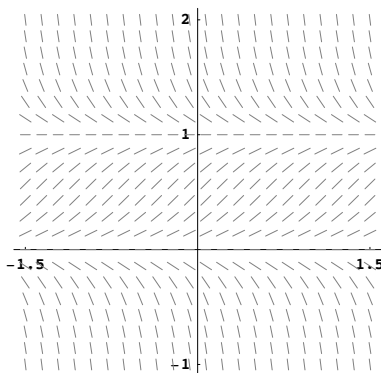
This is a nonautonomous linear differential equation that is related to simple models of voltage in an electric circuit (k , a , and b are parameters).

Comments:

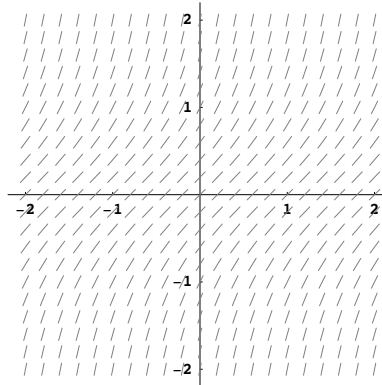
1. Many interesting models in science and engineering are autonomous (but not every model).
2. Every autonomous equation is separable, but the integrals may be impossible to calculate in terms of standard functions.

Basic Fact: Given the graph of one solution to an autonomous equation, we can get the graphs of many other solutions by translating that graph left or right.

Example 1. $\frac{dy}{dt} = 4y(1 - y)$

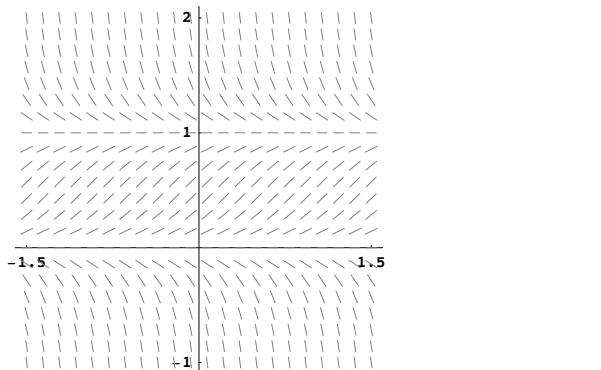


Example 2. $\frac{dy}{dt} = 1 + y^2$



The slope field has so much redundant information that we can replace it with the **phase line**. Here's the phase line for our standard example:

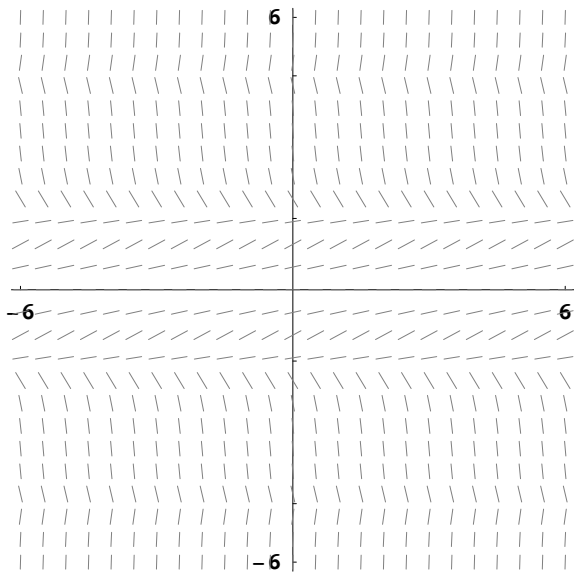
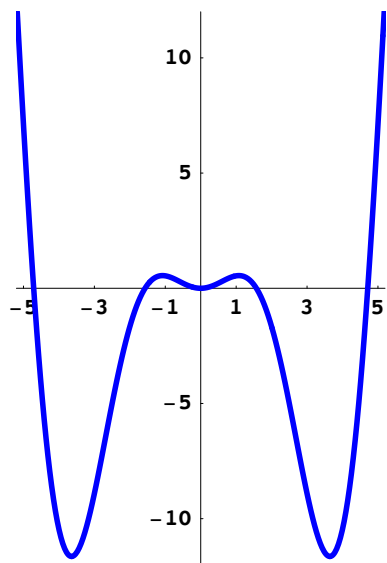
Example. $\frac{dy}{dt} = 4y(1 - y)$



Building phase lines

How do we go about building a phase line from a differential equation?

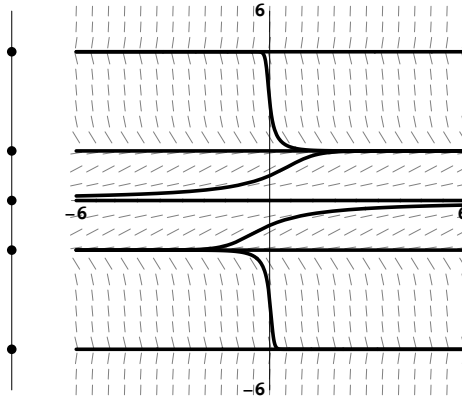
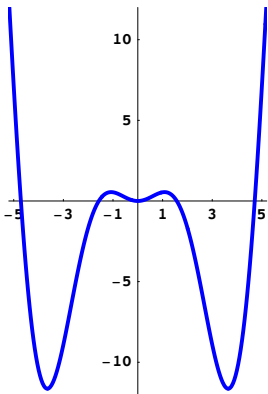
Example $\frac{dy}{dt} = y^2 \cos y$



A little more about phase lines

At the end of last class we discussed the following example.

Example 1. $\frac{dy}{dt} = y^2 \cos y$



Example 2. $\frac{dy}{dt} = f(y)$ where $f(y)$ is given by the graph

