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Math 245

Homework 4

Due on February 12, 2025

Turn in your **best problem** from the three problems below.

You may turn in one problem rated as [2+] or [3-] from Homework 3.

Note that since both this homework and Homework 5 contains fewer problems than usual, you will be allowed to choose one problem from this set for Homework 5 even if it is rated as [2].

1.47ab [2] Let D be the operator $\frac{d}{dx}$.

a. Show that $(xD)^n = \sum_{k=0}^{n} S(n,k)x^k D^k$.

b. Show that

$$x^n D^n = xD(xD-1)(xD-2)\cdots(xD-n+1) = \sum_{k=0}^{n} s(n,k)(xD)^k.$$

1.102c [2+] Generalize Problem **1.102ab** (see last homework set) further to $(x_1 + x_2 + \cdots + x_m)^n$, where $x_i x_j = q_j x_j x_i$ for $i > j$, and where the q_j's are variables commuting with all the x_i's and with each other.

E1 [2] Find transition matrices between the basis $\left\{\binom{n}{i} \ : \ 0 \le i \le d\right\}$ and the basis $\left\{\binom{n+d-i}{d} \ : \ 0 \le i \le d\right\}$.

4.55 [2+] Let $P \subset \mathbb{R}^{2\times 3}$ be the set of 2×3 matrices with nonnegative entries such that every row sums to 3 and every column to 2. Find an explicit formula for $i(P,n)$ and compute (as a rational function reduced to lowest terms) the generating function $\sum_{n\ge 0} i(P,n)x^n$.