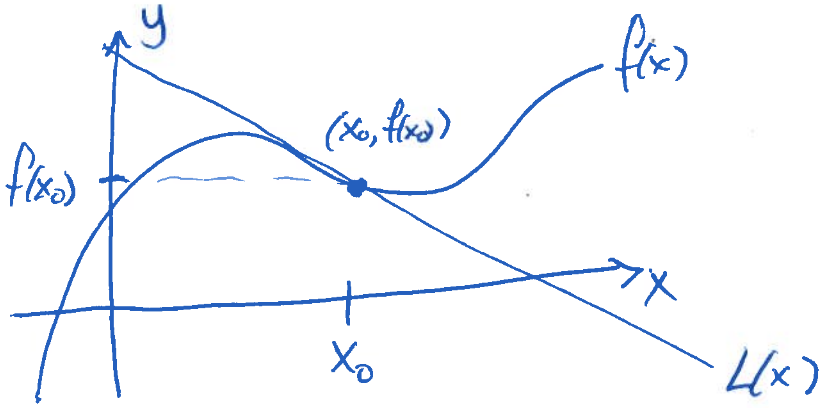
Tangent Planes and Linear Approximations

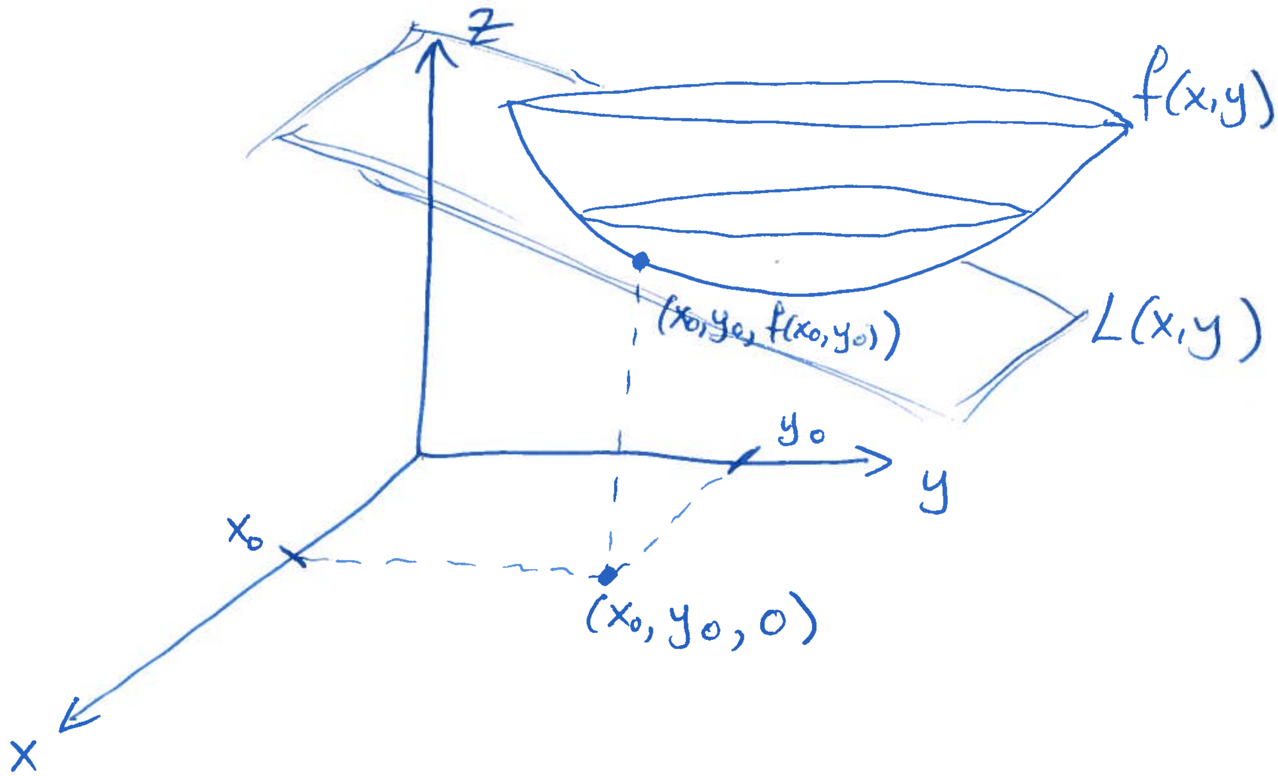
Recall From 17A



near x_0,

$$f(x) \approx L(x) = f'(x_0)(x - x_0) + f(x_0)$$

Now, in 17C

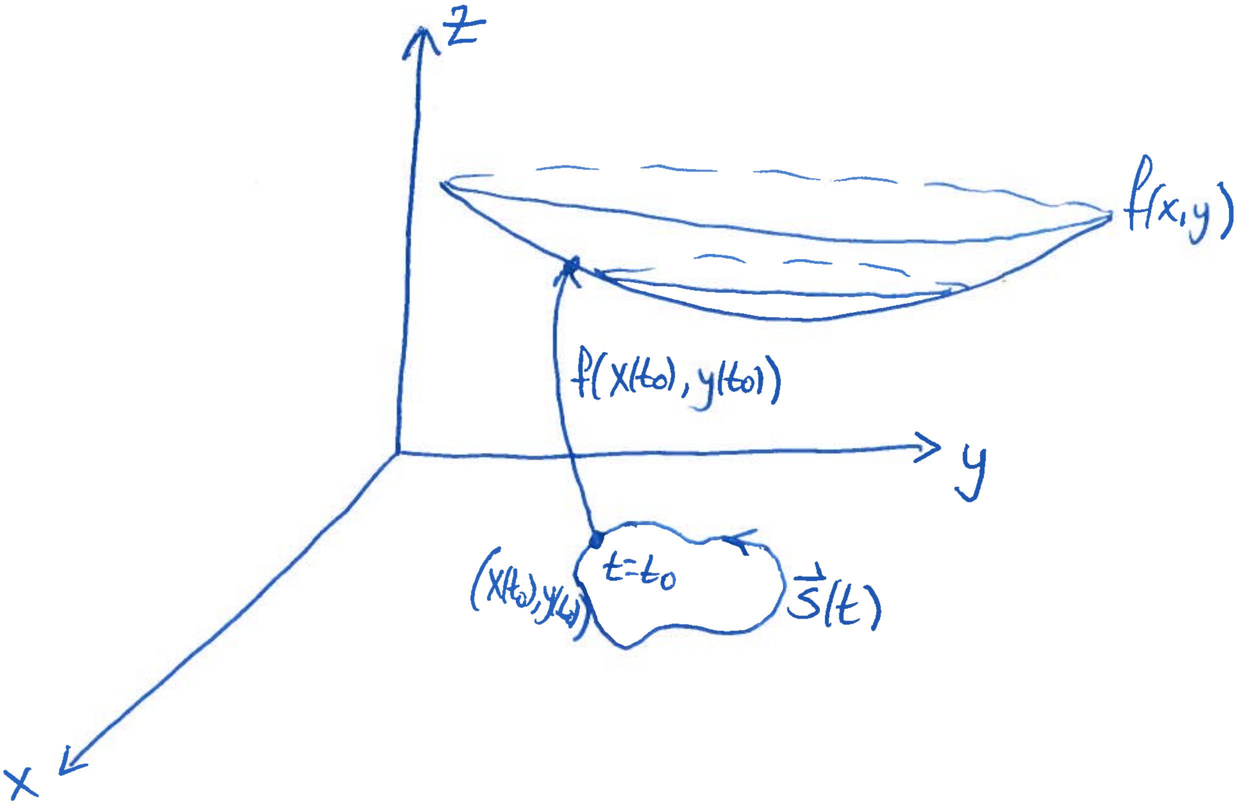


Near (x_0, y_0),

$$f(x,y) \approx L(x,y) = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) + f(x_0,y_0)$$

The Chain Rule

Consider the function $z=f(x,y)$ and the parametric curve $\langle x(t), y(t)\rangle = \vec{s}(t)$.



i.e. The curve in the xy-plane traces out a curve <u>on</u> the surface of $z=f(x,y)$

<u>Q</u>: How does the height change as you travel around the curve?

i.e. What is the rate of change of ~~th~~ your altitude as you travel around the curve?

$\frac{dz}{dt}$! But $z = f(x,y)$ does not directly ~~require~~ depend on t, so we need the chain rule!

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

"How your altitute changes" "$\frac{\Delta x}{\Delta t}$" "$\frac{\Delta y}{\Delta t}$"

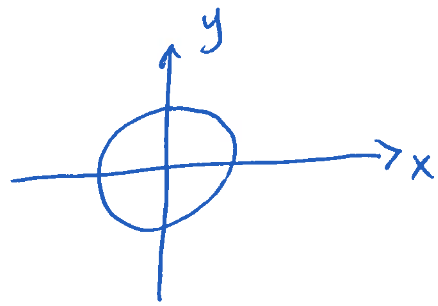
Implicit Differentiation

Recall An implicit function of 2 variables is of the form $F(x,y)=0$.

Eg $F(x,y) = x^2+y^2-4$.

What is this? A circle of radius 2!

$$F(x,y)=0$$
$$x^2+y^2-4=0$$
$$x^2+y^2=4$$



We can compute $\frac{dy}{dx}$ using the formula

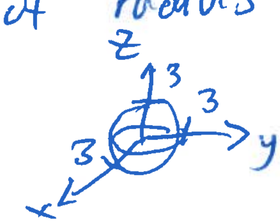
$$\frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

Now, consider $F(x,y,z)=0$. This implicitly defines a surface.

Eg $F(x,y,z) = x^2+y^2+z^2-9$

What is this? A sphere of radius 3!

$$F(x,y,z)=0$$
$$x^2+y^2+z^2-9=0$$
$$x^2+y^2+z^2=9$$



We can compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ using the formulas

$$\frac{\partial z}{\partial x} = - \frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} \qquad \frac{\partial z}{\partial y} = - \frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$$