# Qualitative Analyses of Linear Systems

2D System of differential equations

$$\frac{d\vec{x}}{dt} = A(t)\vec{x} + \vec{g}(t) \quad , \quad \vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$A$ is a matrix
$\vec{g}$ is a vector
$\vec{x}$ is a vector

<u>Classifications</u>

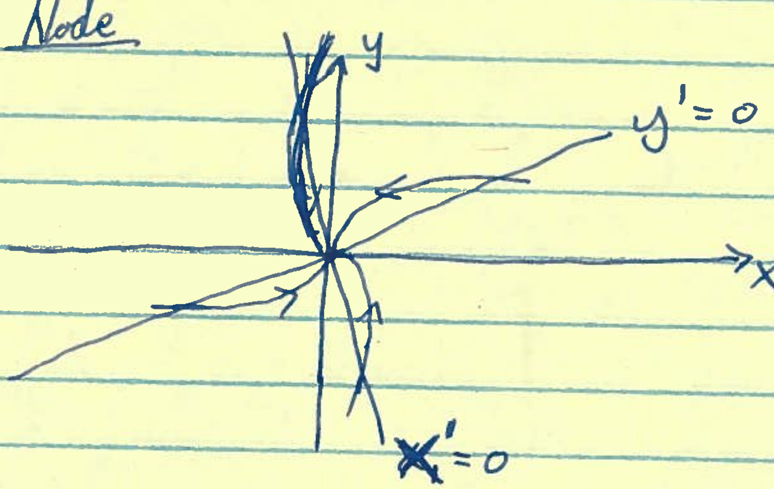
1. If $A$ and $g$ are independent of $t$, then system is called autonomous
   → Otherwise nonautonomous
2. If $\vec{g} = 0$, then system is called homogeneous
   → Otherwise, nonhomogeneous

<u>Qualitative Features</u>

- Equilibria – points where $x'(t) = 0$ <u>and</u> $y'(t) = 0$
- Nullclines – Curves where $x'(t) = 0$ <u>or</u> $y'(t) = 0$
- Stability – classification of equilibria
  - Stable – if you start near the equilibria, you end up there as $t \to \infty$
  - Unstable – not stable
- Types of equilibria – Saddles, nodes, spirals

## Saddle

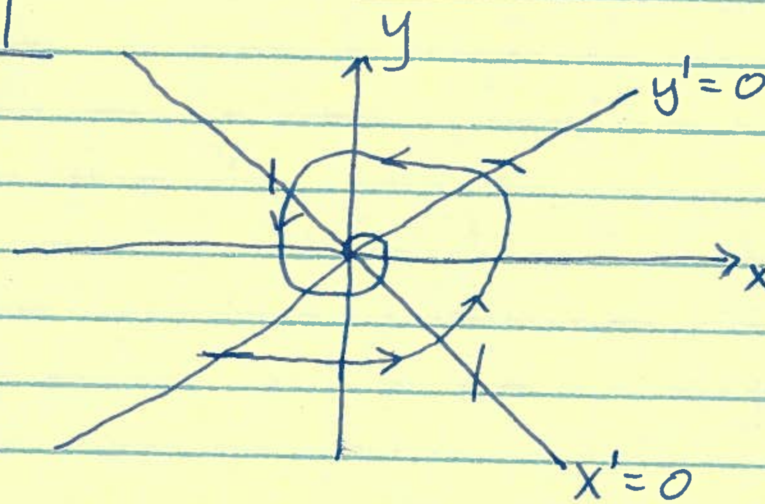

## Node




(Stable node)

- Unstable node will have arrows reversed

## Spiral




(Stable spiral)

- Unstable spiral will have arrows reversed.