Metapopulations

x_A - Subpopulation 1

x_B - Subpopulation 2

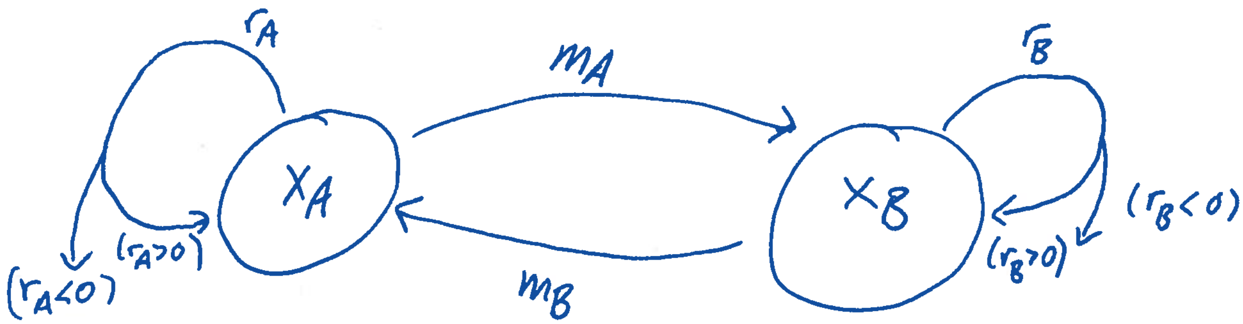
$$\frac{dx_A}{dt} = r_A x_A - m_A x_A + m_B x_B$$

$$\frac{dx_B}{dt} = r_B x_B - m_B x_B + m_A x_A$$

r_i - per capita growth rate

m_i - per capita ~~growth~~ movement rate

Compartment ~~Model~~ Diagram



Ex: x_A - Turkeys that live on campus

x_B - Turkeys that live off campus (in Davis)

Transport of Environmental Pollutants

$$\frac{dl_1}{dt} = -r_{12}\frac{l_1}{V} + r_{31}\frac{l_3}{V}$$

$$\frac{dl_2}{dt} = -r_{23}\frac{l_2}{V} + r_{12}\frac{l_1}{V}$$

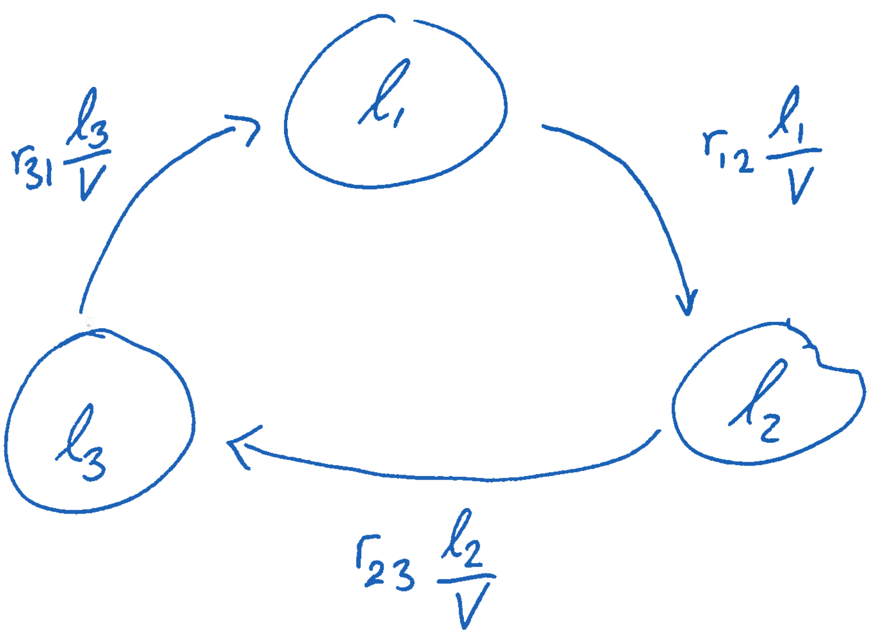
$$\frac{dl_3}{dt} = -r_{31}\frac{l_3}{V} + r_{23}\frac{l_2}{V}$$

$l_i(t)$ - mass of pollutant in lake at time t

r_{ij} - flow from lake i to lake j

V - volume of lake

Compartment diagram



Nonhomogeneous to homogeneous Transformation

Consider the linear autonomous system

$$\vec{x}' = A\vec{x} + \vec{g} \quad (1)$$

Define a new variable

$$\tilde{\vec{x}} = \vec{x} + A^{-1}\vec{g} \quad (2)$$

Then,

$$\tilde{\vec{x}}' = \vec{x}' \quad (3)$$

Now, we hope that $\tilde{\vec{x}}$ solves the homogeneous equation

$$\tilde{\vec{x}}' = A\tilde{\vec{x}}. \quad (4)$$

Using (2) in (4), we have

$$\begin{aligned} A\tilde{\vec{x}} &= A(\vec{x} + A^{-1}\vec{g}) \\ &= A\vec{x} + \vec{g} \\ &= \vec{x}' \quad \text{(by (1))} \end{aligned}$$

From (3), we know

$$\tilde{\vec{x}}\,' = \vec{x}\,'$$

so

$$\tilde{\vec{x}}\,' = A\tilde{\vec{x}}$$

and so

$$\tilde{\vec{x}} = \vec{x} + A^{-1}\vec{g}$$

is a solution to the homogeneous system

$$\tilde{\vec{x}}\,' = A\tilde{\vec{x}} \,.$$