Counting

Fundamental Principle of Counting

- K independent selection
- i^{th} ~~choice~~ selection has n_i choices

Total number of possible choices is

$$n_1 \cdot n_2 \cdot \cdots \cdot n_K$$

Permutations

"order matters"

- ordered arrangement of objects
- Number of permutations of n objects is

$$P_n = n!$$

- Number of permutations of n objects taken K at a time is

$$P_{n,K} = \frac{n!}{(n-K)!}$$

Combination

"order does not matter"

- collection of unordered objects
- Number of combinations of a set of n objects taken K at a time is

$$C_{n,K} = \frac{n!}{K!(n-K)!} = \underbrace{\binom{n}{K}}_{\text{binomial coefficient}}$$

Probability

Experiment with outcomes

- Each instance of an experiment is a trial

Sample space Ω

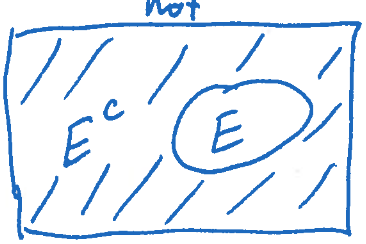
- collection of all possible outcomes

Event

- subset of Ω
- simple → single outcome
- compound → more than one outcome

Sets

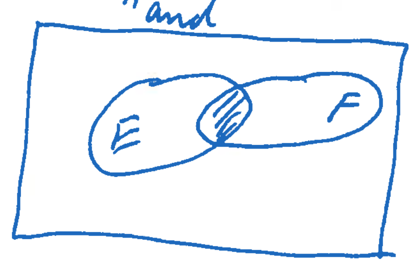
Complement
"not"



Union
"or"

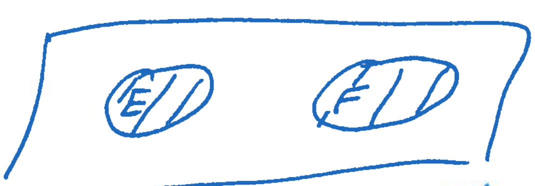
$E \cup F$

Intersection
"and"



$E \cap F$

Disjoint



$E \cap F = \emptyset$

- $\emptyset$ is the empty set

Back to probability...

- Any event E must have $0 \le P(E) \le 1$
- Ω sample space, $P(\Omega) = 1$.

Intuitive notion of probability:

$$P(E) = \frac{n(E)}{n(\Omega)}$$

$n(E)$ ← number of way to have event occur

$n(\Omega)$ ← number of total possible outcomes

- drawing from a deck, rolling a die, flipping a coin

$E \cap F = \emptyset$

$$P(E \cup F) = P(E) + P(F) \qquad (\star)$$

E^c

$$P(E^c) = 1 - P(E)$$

$E \cup F$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

"Need to subtract overlap"

- Notice similarity to $(\star)$