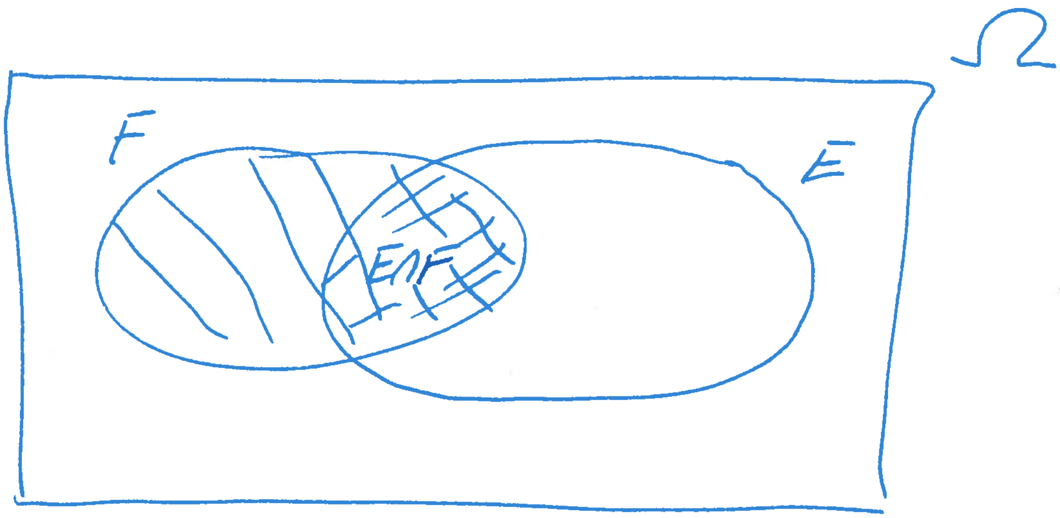
Conditional Probability

Consider events E and F with $P(F)$. The probability of E given F is

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$



"We know F happened so it is now our universe"

→ Only way for E to happen is when $E \cap F$

Complement rule for conditional probability

$$P(E^c|F) = 1 - P(E|F)$$

Multiplication Rule and Independence

multiplication rule: $P(E \cap F) = P(E|F)\,P(F)$

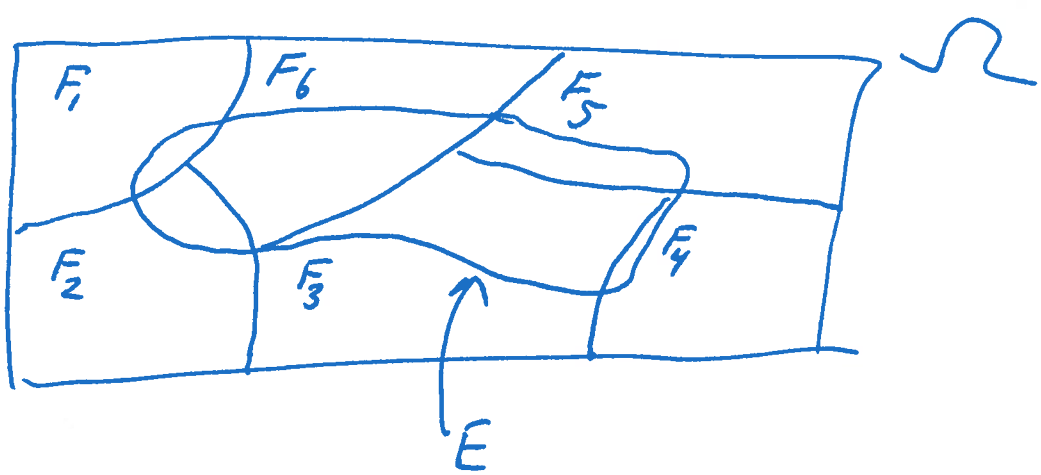
Independence

E and F independent if $P(E \cap F) = P(E)P(F)$

Law of Total Probability

$F_1, F_2, \ldots, F_k$ pairwise disjoint, exhaustive events. Then

$$P(E) = P(E|F_1)P(F_1) + \cdots + P(E|F_k)P(F_k)$$



Bayes' Rule

$$P(F|E) = \frac{P(E|F)\,P(F)}{P(E)}$$

Discrete Random Variables

- a way to assign events to numbers
- X takes in an ~~event~~ element of sample space and ~~gives~~ outputs a number
- "$X=i$" means a particular element of sample space taking place

  $\rightarrow P(X=i)$ is the probability of that outcome

Probability density function (PDF)

$$p_i = P(X=i)$$

$\uparrow$
PDF

- $p_i \geq 0, \ \forall i$
- $\sum_i p_i = 1$

Cumulative distribution function (CDF)

$$F_i = P(X \leq i)$$

"what is the probability of events up to i occurring?"

Mean and Variance of Discrete Random Variables

Expected value / mean (center of mass) : $\mathbb{E}[X] = \sum_i i p_i$

"What are you expected winnings?"

Variance : $Var[X] = \sum_i (i - \mathbb{E}[X])^2 p_i$

"~how far do go away from the mean?"

- Standard deviation : $\sigma^2 = Var[X]$

Standard deviation

Bernoulli Random Variables

- takes values in $\{0, 1\}$

* $P(X=1) = p$

  $P(X=0) = 1-p$