Math 135A, Fall 2025.

Homework 4

1. A casino offers the following game. Toss two coins. If you toss two Heads, you win \$2, if you toss two Tails, you win \$1, otherwise (if the outcomes of the two tosses are different), you win \$$-2$, i.e., you lose \$2. Let X be the amount you win. Compute the p.m.f. of X and your expected winnings EX.

2. A bag contains 8 white, 4 black and 2 grey balls. You choose two balls at random from a bag, without replacement, and win \$2 for each black ball but lose \$1 for each white ball. Let X denote your winnings. Compute the p.m.f. of X. Compute EX.

3. Let X be the difference between the number of Heads and the number of Tails in n tosses of a fair coin. Compute the p.m.f. of X.

You should also do the Problem 3 in Section 5 of the book.

Solutions

1. We have

$$P(X=2)=1/4$$
$$P(X=1)=1/4$$
$$P(X=-2)=1/2$$

and so

$$EX=2\cdot P(X=2)+1\cdot P(X=1)-2\cdot P(X=-2)=-1/4.$$

2. The number of outcomes is $\binom{14}{2}=91$. Then

$$P(X=4)=\binom{4}{2}/\binom{14}{2}=\frac{6}{91}$$
$$P(X=2)=4\cdot 2/\binom{14}{2}=\frac{8}{91}$$
$$P(X=1)=4\cdot 8/\binom{14}{2}=\frac{32}{91}$$
$$P(X=0)=1/\binom{14}{2}=\frac{1}{91}$$
$$P(X=-1)=8\cdot 2/\binom{14}{2}=\frac{16}{91}$$
$$P(X=-2)=\binom{8}{2}/\binom{14}{2}=\frac{28}{91}$$

and so

$$EX=4\cdot P(X=4)+2\cdot P(X=2)+P(X=1)-P(X=-1)-2\cdot P(X=-2)=0,$$

as it must be because the payoff for the black ball is twice the negative payoff for white ball, which is balanced by the twice higher probability for the white ball.

3. To compute $P(X=i)$, the number of Heads h and number of Tails t must satisfy $h+t=n$ and $h-t=i$, so that $t=(n-i)/2$. It follows that $n-i$ must be even and that

$$P(X=i)=\frac{\binom{n}{(n-i)/2}}{2^n},$$

if $n-i$ is even and $-n\le i\le n$.