Math 16C, Winter 1997.
Mar. 3, 1997.

MIDTERM EXAM 2

NAME(print): KEY

NAME(sign):

ID#:

Instructions: Each of the four problems is worth 25 points. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor in determining credit. Calculators, books or notes are not allowed.

Make sure that you have a total of 5 pages (including this one) with 4 problems. Read through the entire exam before beginning to work.

1	
2	
3	
4	
TOTAL	

1. Find and classify all the critical points of the function

$$f(x,y) = \frac{1}{8}x^4 + y^2 - x^2 - 2y.$$

$f_x = \frac{1}{8} \cdot 4x^3 - 2x = \frac{1}{2}x(x^2-4) = 0, \; x = 0, \pm 2$

$f_y = 2y - 2 = 2(y-1) = 0, \; y = 1$

3 critical points:

$(0,1)$ $(2,1)$ and $(-2,1)$.

	$(0,1)$	$(2,1)$	$(-2,1)$
$f_{xx} = \frac{3}{2}x^2 - 2$	-2	4	4
$f_{yy} = 2$	2	2	2
$f_{xy} = 0$	0	0	0
$d = f_{xx} f_{yy} - f_{xy}^2$	-4	8	8
	↑ saddle	↑ local min	↑ local min

2. Use Lagrange multipliers (or any other correct method) to find the maximum of $f(x,y) = e^{xy}$ on the circle $x^2 + y^2 = 8$.

$$F = e^{xy} - \lambda(x^2+y^2-8)$$

$$\frac{\partial F}{\partial x} = e^{xy}\cdot y - 2\lambda x = 0 \qquad 2\lambda = \frac{e^{xy}\cdot y}{x}$$

$$\frac{\partial F}{\partial y} = e^{xy}\cdot x - 2\lambda y = 0 \qquad 2\lambda = \frac{e^{xy}\cdot x}{y}$$

$$\frac{\partial F}{\partial x} = -(x^2+y^2-8) = 0$$

$$\frac{\cancel{e^{xy}}\cdot y}{x} = \frac{\cancel{e^{xy}}\cdot x}{y} \qquad y^2 = x^2$$

$x^2 = 4, \quad x = \pm 2$

$y^2 = 4, \quad y = \pm 2$

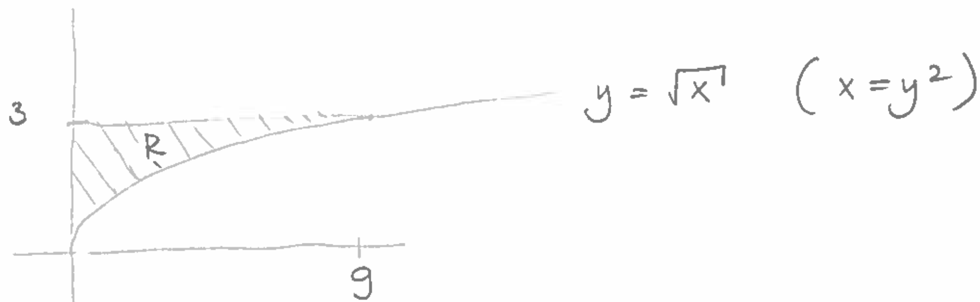
	e^{xy}
$(-2,-2)$	e^4
$(-2,2)$	e^{-4}
$(2,-2)$	e^4
$(2,2)$	e^{-4}

Maximum is e^4.

3.

(a) Evaluate the following double integral (sketch the region and switch the order of integration, if necessary):

$$\int_0^9 dx \left[\int_{\sqrt{x}}^3 \cos(y^3)\,dy\right] dx$$



$$= \int_0^3 \left[\int_0^{y^2} \cos(y^3)\,dx \right] dy =$$

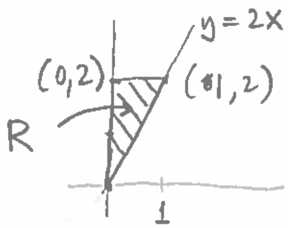
$$= \int_0^3 \cos(y^3) \cdot y^2\,dy = \frac{1}{3}\int_0^{27} \cos u \,du = \frac{1}{3} \sin u \Big|_{u=0}^{u=27}$$

$y^3 = u$

$3y^2\,dy = du$

$$= \frac{\sin 27}{3}$$

(b) Find the volume of the solid which is bounded above by the surface $z = x^2 + 3y^2$ and below by the triangle in the xy plane with vertices $(0,0)$, $(0,2)$ and $(1,2)$.



$$\iint_R (x^2 + 3y^2)\,dA =$$

$$= \int_0^1 \left[\int_{2x}^2 (x^2 + 3y^2)\,dy \right] dx$$

$$= \int_0^1 \left[x^2 y + y^3 \right]_{y=2x}^{y=2} dx$$

$$= \int_0^1 \left[-2x^3 - 8x^3 + 2x^2 + 8\right] dx = \int_0^1 \left[8 + 2x^2 - 10x^3\right] dx = 8 + \frac{2}{3} - \frac{5}{2}$$

$$= \boxed{\frac{37}{6}}$$

4.

(a) Compute: $\sum_{n=1}^{\infty} \frac{2^{n-1}\cancel{3^n}3^{-n}}{5^n}$. $= \sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{2}{15}\right)^n$.

$= \frac{1}{2}\left(\frac{2}{15} + \left(\frac{2}{15}\right)^2 + \cdots\right)$

$= \frac{1}{2} \cdot \frac{2}{15} \cdot \frac{1}{1-\frac{2}{15}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{15}} \cdot \frac{\cancel{15}}{13}$

$= \underline{\underline{\frac{1}{13}}}$

(b) Determine whether the sequence given by $a_n = \dfrac{5 \cdot 3^n + 5^n}{2^n + 6 \cdot 5^n}$ converges or not.

$a_n = \dfrac{5 \cdot \left(\frac{3}{5}\right)^n + 1}{\left(\frac{2}{5}\right)^n + 6}$

(divide top & bottom by 5^n)

$\underline{\underline{\lim_{n\to\infty} a_n = \frac{1}{6}}}$

Converges