9. What exactly does $\lim_{x\to c} f(x) = L$ mean? Give an example in which you find a $\delta > 0$ for a given f, L, c, and $\varepsilon > 0$ in the precise definition of limit.
10. Give precise definitions of the following statements.
    a. $\lim_{x\to 2^-} f(x) = 5$
    b. $\lim_{x\to 2^+} f(x) = 5$
    c. $\lim_{x\to 2} f(x) = \infty$
    d. $\lim_{x\to 2} f(x) = -\infty$
11. What conditions must be satisfied by a function if it is to be continuous at an interior point of its domain? At an endpoint?
12. How can looking at the graph of a function help you tell where the function is continuous?
13. What does it mean for a function to be right-continuous at a point? Left-continuous? How are continuity and one-sided continuity related?
14. What does it mean for a function to be continuous on an interval? Give examples to illustrate the fact that a function that is not continuous on its entire domain may still be continuous on selected intervals within the domain.
15. What are the basic types of discontinuity? Give an example of each. What is a removable discontinuity? Give an example.
16. What does it mean for a function to have the Intermediate Value Property? What conditions guarantee that a function has this property over an interval? What are the consequences for graphing and solving the equation $f(x) = 0$?
17. Under what circumstances can you extend a function $f(x)$ to be continuous at a point $x = c$? Give an example.
18. What exactly do $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to-\infty} f(x) = L$ mean? Give examples.
19. What are $\lim_{x\to\pm\infty} k$ (k a constant) and $\lim_{x\to\pm\infty} (1/x)$? How do you extend these results to other functions? Give examples.
20. How do you find the limit of a rational function as $x \to \pm\infty$? Give examples.
21. What are horizontal and vertical asymptotes? Give examples.

CHAPTER 2 Practice Exercises

Limits and Continuity

1. Graph the function

$$f(x) = \begin{cases} 1, & x \le -1 \\ -x, & -1 < x < 0 \\ 1, & x = 0 \\ -x, & 0 < x < 1 \\ 1, & x \ge 1. \end{cases}$$

Then discuss, in detail, limits, one-sided limits, continuity, and one-sided continuity of f at $x = -1$, 0, and 1. Are any of the discontinuities removable? Explain.

2. Repeat the instructions of Exercise 1 for

$$f(x) = \begin{cases} 0, & x \le -1 \\ 1/x, & 0 < |x| < 1 \\ 0, & x = 1 \\ 1, & x > 1. \end{cases}$$

3. Suppose that $f(t)$ and $f(t)$ are defined for all t and that $\lim_{t\to t_0} f(t) = -7$ and $\lim_{t\to t_0} g(t) = 0$. Find the limit as $t \to t_0$ of the following functions.
    a. $3f(t)$
    b. $(f(t))^2$
    c. $f(t)\cdot g(t)$
    d. $\dfrac{f(t)}{g(t) - 7}$
    e. $\cos(g(t))$
    f. $|f(t)|$
    g. $f(t) + g(t)$
    h. $1/f(t)$
4. Suppose the functions $f(x)$ and $g(x)$ are defined for all x and that $\lim_{x\to 0} f(x) = 1/2$ and $\lim_{x\to 0} g(x) = \sqrt{2}$. Find the limits as $x \to 0$ of the following functions.
    a. $-g(x)$
    b. $g(x)\cdot f(x)$
    c. $f(x) + g(x)$
    d. $1/f(x)$
    e. $x + f(x)$
    f. $\dfrac{f(x)\cdot \cos x}{x - 1}$

In Exercises 5 and 6, find the value that $\lim_{x\to 0} g(x)$ must have if the given limit statements hold.

5. $\lim_{x\to 0}\left(\dfrac{4 - g(x)}{x}\right) = 1$
6. $\lim_{x\to -4}\left(x \lim_{x\to 0} g(x)\right) = 2$
7. On what intervals are the following functions continuous?
    a. $f(x) = x^{1/3}$
    b. $g(x) = x^{3/4}$
    c. $h(x) = x^{-2/3}$
    d. $k(x) = x^{-1/6}$
8. On what intervals are the following functions continuous?
    a. $f(x) = \tan x$
    b. $g(x) = \csc x$
    c. $h(x) = \dfrac{\cos x}{x - \pi}$
    d. $k(x) = \dfrac{\sin x}{x}$

Finding Limits

In Exercises 9–28, find the limit or explain why it does not exist.

9. $\lim \dfrac{x^2 - 4x + 4}{x^3 + 5x^2 - 14x}$
    a. as $x \to 0$
    b. as $x \to 2$
10. $\lim \dfrac{x^2 + x}{x^5 + 2x^4 + x^3}$
    a. as $x \to 0$
    b. as $x \to -1$
11. $\lim_{x\to 1} \dfrac{1 - \sqrt{x}}{1 - x}$
12. $\lim_{x\to a} \dfrac{x^2 - a^2}{x^4 - a^4}$
13. $\lim_{h\to 0} \dfrac{(x + h)^2 - x^2}{h}$
14. $\lim_{x\to 0} \dfrac{(x + h)^2 - x^2}{h}$

15. $\lim_{x\to 0} \dfrac{\dfrac{1}{2+x} - \dfrac{1}{2}}{x}$

16. $\lim_{x\to 0} \dfrac{(2+x)^3 - 8}{x}$

17. $\lim_{x\to 1} \dfrac{x^{1/3} - 1}{\sqrt{x} - 1}$

18. $\lim_{x\to 64} \dfrac{x^{2/3} - 16}{\sqrt{x} - 8}$

19. $\lim_{x\to 0} \dfrac{\tan(2x)}{\tan(\pi x)}$

20. $\lim_{x\to \pi^-} \csc x$

21. $\lim_{x\to \pi} \sin\left(\dfrac{x}{2} + \sin x\right)$

22. $\lim_{x\to \pi} \cos^2(x - \tan x)$

23. $\lim_{x\to 0} \dfrac{8x}{3\sin x - x}$

24. $\lim_{x\to 0} \dfrac{\cos 2x - 1}{\sin x}$

25. $\lim_{t\to 3^+} \ln(t-3)$

26. $\lim_{t\to 1} t^2 \ln(2 - \sqrt{t})$

27. $\lim_{\theta\to 0^+} \sqrt{\theta}\, e^{\cos(\pi/\theta)}$

28. $\lim_{z\to 0^+} \dfrac{2e^{1/z}}{e^{1/z} + 1}$

In Exercises 29–32, find the limit of $g(x)$ as x approaches the indicated value.

29. $\lim_{x\to 0^+} (4g(x))^{1/3} = 2$

30. $\lim_{x\to \sqrt{5}} \dfrac{1}{x + g(x)} = 2$

31. $\lim_{x\to 1} \dfrac{3x^2 + 1}{g(x)} = \infty$

32. $\lim_{x\to -2} \dfrac{5 - x^2}{\sqrt{g(x)}} = 0$

T Roots

33. Let $f(x) = x^3 - x - 1$.

a. Use the Intermediate Value Theorem to show that f has a zero between -1 and 2.

b. Solve the equation $f(x) = 0$ graphically with an error of magnitude at most 10^{-8}.

c. It can be shown that the exact value of the solution in part **(b)** is

$$\left(\frac{1}{2} + \frac{\sqrt{69}}{18}\right)^{1/3} + \left(\frac{1}{2} - \frac{\sqrt{69}}{18}\right)^{1/3}.$$

Evaluate this exact answer and compare it with the value you found in part (b).

T 34. Let $f(\theta) = \theta^3 - 2\theta + 2$.

a. Use the Intermediate Value Theorem to show that f has a zero between -2 and 0.

b. Solve the equation $f(\theta) = 0$ graphically with an error of magnitude at most 10^{-4}.

c. It can be shown that the exact value of the solution in part (b) is

$$\left(\sqrt{\frac{19}{27}} - 1\right)^{1/3} - \left(\sqrt{\frac{19}{27}} + 1\right)^{1/3}.$$

Evaluate this exact answer and compare it with the value you found in part (b).

Continuous Extension

35. Can $f(x) = x(x^2 - 1)/|x^2 - 1|$ be extended to be continuous at $x = 1$ or -1? Give reasons for your answers. (Graph the function—you will find the graph interesting.)

36. Explain why the function $f(x) = \sin(1/x)$ has no continuous extension to $x = 0$.

T In Exercises 37–40, graph the function to see whether it appears to have a continuous extension to the given point a. If it does, use Trace and Zoom to find a good candidate for the extended function's value at a. If the function does not appear to have a continuous extension, can it be extended to be continuous from the right or left? If so, what do you think the extended function's value should be?

37. $f(x) = \dfrac{x - 1}{x - \sqrt[4]{x}}, \quad a = 1$

38. $g(\theta) = \dfrac{5\cos\theta}{4\theta - 2\pi}, \quad a = \pi/2$

39. $h(t) = (1 + |t|)^{1/t}, \quad a = 0$

40. $k(x) = \dfrac{x}{1 - 2^{|x|}}, \quad a = 0$

Limits at Infinity

Find the limits in Exercises 41–54.

41. $\lim_{x\to \infty} \dfrac{2x + 3}{5x + 7}$

42. $\lim_{x\to -\infty} \dfrac{2x^2 + 3}{5x^2 + 7}$

43. $\lim_{x\to -\infty} \dfrac{x^2 - 4x + 8}{3x^3}$

44. $\lim_{x\to \infty} \dfrac{1}{x^2 - 7x + 1}$

45. $\lim_{x\to -\infty} \dfrac{x^2 - 7x}{x + 1}$

46. $\lim_{x\to \infty} \dfrac{x^4 + x^3}{12x^3 + 128}$

47. $\lim_{x\to \infty} \dfrac{\sin x}{\lfloor x \rfloor}$ (If you have a grapher, try graphing the function for $-5 \le x \le 5$.)

48. $\lim_{\theta\to \infty} \dfrac{\cos\theta - 1}{\theta}$ (If you have a grapher, try graphing $f(x) = x(\cos(1/x) - 1)$ near the origin to "see" the limit at infinity.)

49. $\lim_{x\to \infty} \dfrac{x + \sin x + 2\sqrt{x}}{x + \sin x}$

50. $\lim_{x\to \infty} \dfrac{x^{2/3} + x^{-1}}{x^{2/3} + \cos^2 x}$

51. $\lim_{x\to \infty} e^{1/x} \cos\dfrac{1}{x}$

52. $\lim_{t\to \infty} \ln\left(1 + \dfrac{1}{t}\right)$

53. $\lim_{x\to -\infty} \tan^{-1} x$

54. $\lim_{t\to -\infty} e^{3t} \sin^{-1}\dfrac{1}{t}$

Horizontal and Vertical Asymptotes

55. Use limits to determine the equations for all vertical asymptotes.

a. $y = \dfrac{x^2 + 4}{x - 3}$

b. $f(x) = \dfrac{x^2 - x - 2}{x^2 - 2x + 1}$

c. $y = \dfrac{x^2 + x - 6}{x^2 + 2x - 8}$

56. Use limits to determine the equations for all horizontal asymptotes.

a. $y = \dfrac{1 - x^2}{x^2 + 1}$

b. $f(x) = \dfrac{\sqrt{x} + 4}{\sqrt{x + 4}}$

c. $g(x) = \dfrac{\sqrt{x^2 + 4}}{x}$

d. $y = \sqrt{\dfrac{x^2 + 9}{9x^2 + 1}}$

57. Determine the domain and range of $y = \dfrac{\sqrt{16 - x^2}}{x - 2}$.

58. Assume that constants a and b are positive. Find equations for all horizontal and vertical asymptotes for the graph of $y = \dfrac{\sqrt{ax^2 + 4}}{x - b}$.

CHAPTER 2 Additional and Advanced Exercises

T **1. Assigning a value to 0^0** The rules of exponents tell us that $a^0 = 1$ if a is any number different from zero. They also tell us that $0^n = 0$ if n is any positive number.

If we tried to extend these rules to include the case 0^0, we would get conflicting results. The first rule would say $0^0 = 1$, whereas the second would say $0^0 = 0$.

We are not dealing with a question of right or wrong here. Neither rule applies as it stands, so there is no contradiction. We could, in fact, define 0^0 to have any value we wanted as long as we could persuade others to agree.

What value would you like 0^0 to have? Here is an example that might help you to decide. (See Exercise 2 below for another example.)

a. Calculate x^x for $x = 0.1, 0.01, 0.001$, and so on as far as your calculator can go. Record the values you get. What pattern do you see?

b. Graph the function $y = x^x$ for $0 < x \le 1$. Even though the function is not defined for $x \le 0$, the graph will approach the y-axis from the right. Toward what y-value does it seem to be headed? Zoom in to further support your idea.

T **2. A reason you might want 0^0 to be something other than 0 or 1** As the number x increases through positive values, the numbers $1/x$ and $1/(\ln x)$ both approach zero. What happens to the number

$$f(x) = \left(\frac{1}{x}\right)^{1/(\ln x)}$$

as x increases? Here are two ways to find out.

a. Evaluate f for $x = 10, 100, 1000$, and so on as far as your calculator can reasonably go. What pattern do you see?

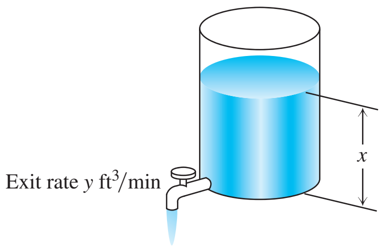
b. Graph f in a variety of graphing windows, including windows that contain the origin. What do you see? Trace the y-values along the graph. What do you find?

3. Lorentz contraction In relativity theory, the length of an object, say a rocket, appears to an observer to depend on the speed at which the object is traveling with respect to the observer. If the observer measures the rocket's length as L_0 at rest, then at speed υ the length will appear to be

$$L = L_0\sqrt{1 - \frac{\upsilon^2}{c^2}}.$$

This equation is the Lorentz contraction formula. Here, c is the speed of light in a vacuum, about 3×10^8 m/sec. What happens to L as υ increases? Find $\lim_{\upsilon \to c^-} L$. Why was the left-hand limit needed?

4. Controlling the flow from a draining tank Torricelli's law says that if you drain a tank like the one in the figure shown, the rate y at which water runs out is a constant times the square root of the water's depth x. The constant depends on the size and shape of the exit valve.



Suppose that $y = \sqrt{x}/2$ for a certain tank. You are trying to maintain a fairly constant exit rate by adding water to the tank with a hose from time to time. How deep must you keep the water if you want to maintain the exit rate

a. within 0.2 ft^3/min of the rate $y_0 = 1$ ft^3/min?

b. within 0.1 ft^3/min of the rate $y_0 = 1$ ft^3/min?

5. Thermal expansion in precise equipment As you may know, most metals expand when heated and contract when cooled. The dimensions of a piece of laboratory equipment are sometimes so critical that the shop where the equipment is made must be held at the same temperature as the laboratory where the equipment is to be used. A typical aluminum bar that is 10 cm wide at 70°F will be

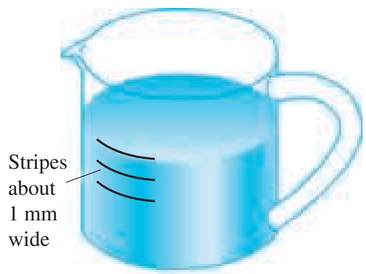
$$y = 10 + (t - 70) \times 10^{-4}$$

centimeters wide at a nearby temperature t. Suppose that you are using a bar like this in a gravity wave detector, where its width must stay within 0.0005 cm of the ideal 10 cm. How close to $t_0 = 70$°F must you maintain the temperature to ensure that this tolerance is not exceeded?

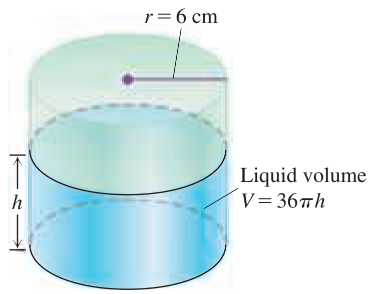
6. Stripes on a measuring cup The interior of a typical 1-L measuring cup is a right circular cylinder of radius 6 cm (see accompanying figure). The volume of water we put in the cup is therefore a function of the level h to which the cup is filled, the formula being

$$V = \pi 6^2 h = 36\pi h.$$

How closely must we measure h to measure out 1 L of water (1000 cm^3) with an error of no more than 1% (10 cm^3)?



(a)



(b)

A 1-L measuring cup (a), modeled as a right circular cylinder (b) of radius $r = 6$ cm

Precise Definition of Limit

In Exercises 7–10, use the formal definition of limit to prove that the function is continuous at c.

7. $f(x) = x^2 - 7, \quad c = 1$ **8.** $g(x) = 1/(2x), \quad c = 1/4$

9. $h(x) = \sqrt{2x - 3}, \quad c = 2$ **10.** $F(x) = \sqrt{9 - x}, \quad c = 5$

11. Uniqueness of limits Show that a function cannot have two different limits at the same point. That is, if $\lim_{x \to c} f(x) = L_1$ and $\lim_{x \to c} f(x) = L_2$, then $L_1 = L_2$.

12. Prove the limit Constant Multiple Rule:

$$\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x) \quad \text{for any constant } k.$$

13. One-sided limits If $\lim_{x \to 0^+} f(x) = A$ and $\lim_{x \to 0^-} f(x) = B$, find

a. $\lim_{x \to 0^+} f(x^3 - x)$ **b.** $\lim_{x \to 0^-} f(x^3 - x)$

c. $\lim_{x \to 0^+} f(x^2 - x^4)$ **d.** $\lim_{x \to 0^-} f(x^2 - x^4)$

14. Limits and continuity Which of the following statements are true, and which are false? If true, say why; if false, give a counterexample (that is, an example confirming the falsehood).

a. If $\lim_{x \to c} f(x)$ exists but $\lim_{x \to c} g(x)$ does not exist, then $\lim_{x \to c}(f(x) + g(x))$ does not exist.

b. If neither $\lim_{x \to c} f(x)$ nor $\lim_{x \to c} g(x)$ exists, then $\lim_{x \to c} (f(x) + g(x))$ does not exist.

c. If f is continuous at x, then so is $|f|$.

d. If $|f|$ is continuous at c, then so is f.

In Exercises 15 and 16, use the formal definition of limit to prove that the function has a continuous extension to the given value of x.

15. $f(x) = \dfrac{x^2 - 1}{x + 1}, \quad x = -1$ **16.** $g(x) = \dfrac{x^2 - 2x - 3}{2x - 6}, \quad x = 3$

17. A function continuous at only one point Let

$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

a. Show that f is continuous at $x = 0$.

b. Use the fact that every nonempty open interval of real numbers contains both rational and irrational numbers to show that f is not continuous at any nonzero value of x.

18. The Dirichlet ruler function If x is a rational number, then x can be written in a unique way as a quotient of integers m/n where $n > 0$ and m and n have no common factors greater than 1. (We say that such a fraction is in *lowest terms*. For example, $6/4$ written in lowest terms is $3/2$.) Let $f(x)$ be defined for all x in the interval $[0, 1]$ by

$$f(x) = \begin{cases} 1/n, & \text{if } x = m/n \text{ is a rational number in lowest terms} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

For instance, $f(0) = f(1) = 1$, $f(1/2) = 1/2$, $f(1/3) = f(2/3) = 1/3$, $f(1/4) = f(3/4) = 1/4$, and so on.

a. Show that f is discontinuous at every rational number in $[0, 1]$.

b. Show that f is continuous at every irrational number in $[0, 1]$. (*Hint*: If ε is a given positive number, show that there are only finitely many rational numbers r in $[0, 1]$ such that $f(r) \geq \varepsilon$.)

c. Sketch the graph of f. Why do you think f is called the "ruler function"?

19. Antipodal points Is there any reason to believe that there is always a pair of antipodal (diametrically opposite) points on Earth's equator where the temperatures are the same? Explain.

20. If $\lim_{x \to c} (f(x) + g(x)) = 3$ and $\lim_{x \to c} (f(x) - g(x)) = -1$, find $\lim_{x \to c} f(x)g(x)$.

21. Roots of a quadratic equation that is almost linear The equation $ax^2 + 2x - 1 = 0$, where a is a constant, has two roots if $a > -1$ and $a \neq 0$, one positive and one negative:

$$r_+(a) = \frac{-1 + \sqrt{1 + a}}{a}, \qquad r_-(a) = \frac{-1 - \sqrt{1 + a}}{a},$$

a. What happens to $r_+(a)$ as $a \to 0$? As $a \to -1^+$?

b. What happens to $r_-(a)$ as $a \to 0$? As $a \to -1^+$?

c. Support your conclusions by graphing $r_+(a)$ and $r_-(a)$ as functions of a. Describe what you see.

d. For added support, graph $f(x) = ax^2 + 2x - 1$ simultaneously for $a = 1, 0.5, 0.2, 0.1,$ and 0.05.

22. Root of an equation Show that the equation $x + 2\cos x = 0$ has at least one solution.

23. Bounded functions A real-valued function f is **bounded from above** on a set D if there exists a number N such that $f(x) \leq N$ for all x in D. We call N, when it exists, an **upper bound** for f on D and say that f is bounded from above by N. In a similar manner, we say that f is **bounded from below** on D if there exists a number M such that $f(x) \geq M$ for all x in D. We call M, when it exists, a **lower bound** for f on D and say that f is bounded from below by M. We say that f is **bounded** on D if it is bounded from both above and below.

a. Show that f is bounded on D if and only if there exists a number B such that $|f(x)| \leq B$ for all x in D.

b. Suppose that f is bounded from above by N. Show that if $\lim_{x \to c} f(x) = L$, then $L \leq N$.

c. Suppose that f is bounded from below by M. Show that if $\lim_{x \to c} f(x) = L$, then $L \geq M$.

24. Max $\{a, b\}$ and min $\{a, b\}$

a. Show that the expression

$$\max\{a, b\} = \frac{a + b}{2} + \frac{|a - b|}{2}$$

equals a if $a \geq b$ and equals b if $b \geq a$. In other words, $\max\{a, b\}$ gives the larger of the two numbers a and b.

b. Find a similar expression for $\min\{a, b\}$, the smaller of a and b.

Generalized Limits Involving $\dfrac{\sin \theta}{\theta}$

The formula $\lim_{\theta \to 0}(\sin \theta)/\theta = 1$ can be generalized. If $\lim_{x \to c} f(x) = 0$ and $f(x)$ is never zero in an open interval containing the point $x = c$, except possibly at c itself, then

$$\lim_{x \to c} \frac{\sin f(x)}{f(x)} = 1.$$

Here are several examples.

a. $\displaystyle \lim_{x \to 0} \frac{\sin x^2}{x^2} = 1$

b. $\displaystyle \lim_{x \to 0} \frac{\sin x^2}{x} = \lim_{x \to 0} \frac{\sin x^2}{x^2} \lim_{x \to 0} \frac{x^2}{x} = 1 \cdot 0 = 0$

c. $$\lim_{x\to-1}\frac{\sin(x^2-x-2)}{x+1}=\lim_{x\to-1}\frac{\sin(x^2-x-2)}{(x^2-x-2)}\cdot$$
$$\lim_{x\to-1}\frac{(x^2-x-2)}{x+1}=1\cdot\lim_{x\to-1}\frac{(x+1)(x-2)}{x+1}=-3$$

d. $$\lim_{x\to1}\frac{\sin\left(1-\sqrt{x}\right)}{x-1}=\lim_{x\to1}\frac{\sin\left(1-\sqrt{x}\right)}{1-\sqrt{x}}\frac{1-\sqrt{x}}{x-1}$$
$$=\lim_{x\to1}\frac{\left(1-\sqrt{x}\right)\left(1+\sqrt{x}\right)}{(x-1)\left(1+\sqrt{x}\right)}$$
$$=\lim_{x\to1}\frac{1-x}{(x-1)\left(1+\sqrt{x}\right)}=-\frac{1}{2}$$

Find the limits in Exercises 25–30.

25. $\lim_{x\to0}\frac{\sin(1-\cos x)}{x}$

26. $\lim_{x\to0^+}\frac{\sin x}{\sin\sqrt{x}}$

27. $\lim_{x\to0}\frac{\sin(\sin x)}{x}$

28. $\lim_{x\to0}\frac{\sin(x^2+x)}{x}$

29. $\lim_{x\to2}\frac{\sin(x^2-4)}{x-2}$

30. $\lim_{x\to9}\frac{\sin\left(\sqrt{x}-3\right)}{x-9}$

Oblique Asymptotes

Find all possible oblique asymptotes in Exercises 31–34.

31. $y=\frac{2x^{3/2}+2x-3}{\sqrt{x}+1}$

32. $y=x+x\sin\frac{1}{x}$

33. $y=\sqrt{x^2+1}$

34. $y=\sqrt{x^2+2x}$

Showing an Equation Is Solvable

35. Assume that $1<a<b$ and $\frac{a}{x}+x=\frac{1}{x-b}$. Show that this equation is solvable.

More Limits

36. Find constants a and b so that each of the following limits is true.

a. $\lim_{x\to0}\frac{\sqrt{a+bx}-1}{x}=2$ **b.** $\lim_{x\to1}\frac{\tan(ax-a)+b-2}{x-1}=3$

37. Evaluate $\lim_{x\to1}\frac{x^{2/3}-1}{1-\sqrt{x}}$.

38. Evaluate $\lim_{x\to0}\frac{|3x+4|-|x|-4}{x}$.

Limits on Arbitrary Domains

The definition of the limit of a function at $x=c$ extends to functions whose domains near c are more complicated than intervals.

> ### General Definition of Limit
> Suppose every open interval containing c contains a point other than c in the domain of f. We say that $\lim_{x\to c}f(x)=L$ if for every number $\varepsilon>0$ there exists a corresponding number $\delta>0$ such that for all x in the domain of f, $|f(x)-L|<\varepsilon$ whenever $0<|x-c|<\delta$.

For the functions in Exercises 39–42,

a. Find the domain.

b. Show that at $c=0$ the domain has the property described above.

c. Evaluate $\lim_{x\to0}f(x)$.

39. The function f is defined as follows: $f(x)=x$ if $x=1/n$ where n is a positive integer, and $f(0)=1$.

40. The function f is defined as follows: $f(x)=1-x$ if $x=1/n$ where n is a positive integer, and $f(0)=1$.

41. $f(x)=\sqrt{x\sin(1/x)}$

42. $f(x)=\sqrt{\ln(\sin(1/x))}$

43. Let g be a function with domain the rational numbers, defined by
$$g(x)=\frac{2}{x-\sqrt{2}}\quad\text{for rational } x.$$

a. Sketch the graph of g as well as you can, keeping in mind that g is only defined at rational points.

b. Use the general definition of a limit to prove that $\lim_{x\to0}g(x)=-\sqrt{2}$.

c. Prove that g is continuous at the point $x=0$ by showing that the limit in part (b) equals $g(0)$.

d. Is g continuous at other points of its domain?

CHAPTER 2 Technology Application Projects

Mathematica/Maple Projects

Projects can be found within MyMathLab.

Take It to the Limit
Part I
Part II (Zero Raised to the Power Zero: What Does It Mean?)
Part III (One-Sided Limits)
Visualize and interpret the limit concept through graphical and numerical explorations.
Part IV (What a Difference a Power Makes)
See how sensitive limits can be with various powers of x.

Going to Infinity
Part I (Exploring Function Behavior as $x\to\infty$ or $x\to-\infty$)
This module provides four examples to explore the behavior of a function as $x\to\infty$ or $x\to-\infty$.
Part II (Rates of Growth)
Observe graphs that *appear* to be continuous, yet the function is not continuous. Several issues of continuity are explored to obtain results that you may find surprising.