

Differentiating both sides with respect to t gives

$$2x \frac{dx}{dt} = 2(25 + h) \frac{dh}{dt},$$

and solving this last equation for dh/dt we find

$$\frac{dh}{dt} = \frac{x}{25 + h} \frac{dx}{dt}. \quad (2)$$

Since we know dx/dt , it remains only to find $25 + h$ at the instant when $x = 21$. From Equation (1),

$$20^2 + 21^2 = (25 + h)^2$$

so that

$$(25 + h)^2 = 841, \quad \text{or} \quad 25 + h = 29.$$

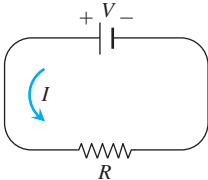
Equation (2) now gives

$$\frac{dh}{dt} = \frac{21}{29} \cdot 4 = \frac{84}{29} \approx 2.9 \text{ ft/sec}$$

as the rate at which the weight is being raised when $x = 21$ ft. ■

EXERCISES 3.10

- Area** Suppose that the radius r and area $A = \pi r^2$ of a circle are differentiable functions of t . Write an equation that relates dA/dt to dr/dt .
- Surface area** Suppose that the radius r and surface area $S = 4\pi r^2$ of a sphere are differentiable functions of t . Write an equation that relates dS/dt to dr/dt .
- Assume that $y = 5x$ and $dx/dt = 2$. Find dy/dt .
- Assume that $2x + 3y = 12$ and $dy/dt = -2$. Find dx/dt .
- If $y = x^2$ and $dx/dt = 3$, then what is dy/dt when $x = -1$?
- If $x = y^3 - y$ and $dy/dt = 5$, then what is dx/dt when $y = 2$?
- If $x^2 + y^2 = 25$ and $dx/dt = -2$, then what is dy/dt when $x = 3$ and $y = -4$?
- If $x^2 y^3 = 4/27$ and $dy/dt = 1/2$, then what is dx/dt when $x = 2$?
- If $L = \sqrt{x^2 + y^2}$, $dx/dt = -1$, and $dy/dt = 3$, find dL/dt when $x = 5$ and $y = 12$.
- If $r + s^2 + v^3 = 12$, $dr/dt = 4$, and $ds/dt = -3$, find dv/dt when $r = 3$ and $s = 1$.
- If the original 24 m edge length x of a cube decreases at the rate of 5 m/min, when $x = 3$ m at what rate does the cube's
 - surface area change?
 - volume change?
- A cube's surface area increases at the rate of 72 in²/sec. At what rate is the cube's volume changing when the edge length is $x = 3$ in?
- Volume** The radius r and height h of a right circular cylinder are related to the cylinder's volume V by the formula $V = \pi r^2 h$.
 - How is dV/dt related to dh/dt if r is constant?
 - How is dV/dt related to dr/dt if h is constant?
 - How is dV/dt related to dr/dt and dh/dt if neither r nor h is constant?
- Volume** The radius r and height h of a right circular cone are related to the cone's volume V by the equation $V = (1/3)\pi r^2 h$.
 - How is dV/dt related to dh/dt if r is constant?
 - How is dV/dt related to dr/dt if h is constant?
 - How is dV/dt related to dr/dt and dh/dt if neither r nor h is constant?
- Changing voltage** The voltage V (volts), current I (amperes), and resistance R (ohms) of an electric circuit like the one shown here are related by the equation $V = IR$. Suppose that V is increasing at the rate of 1 volt/sec while I is decreasing at the rate of 1/3 amp/sec. Let t denote time in seconds.



 - What is the value of dV/dt ?
 - What is the value of dI/dt ?
 - What equation relates dR/dt to dV/dt and dI/dt ?
 - Find the rate at which R is changing when $V = 12$ volts and $I = 2$ amps. Is R increasing, or decreasing?

16. Electrical power The power P (watts) of an electric circuit is related to the circuit's resistance R (ohms) and current I (amperes) by the equation $P = RI^2$.

a. How are dP/dt , dR/dt , and dI/dt related if none of P , R , and I are constant?

b. How is dR/dt related to dI/dt if P is constant?

17. Distance Let x and y be differentiable functions of t and let $s = \sqrt{x^2 + y^2}$ be the distance between the points $(x, 0)$ and $(0, y)$ in the xy -plane.

a. How is ds/dt related to dx/dt if y is constant?

b. How is ds/dt related to dx/dt and dy/dt if neither x nor y is constant?

c. How is dx/dt related to dy/dt if s is constant?

18. Diagonals If x , y , and z are lengths of the edges of a rectangular box, the common length of the box's diagonals is $s = \sqrt{x^2 + y^2 + z^2}$.

a. Assuming that x , y , and z are differentiable functions of t , how is ds/dt related to dx/dt , dy/dt , and dz/dt ?

b. How is ds/dt related to dy/dt and dz/dt if x is constant?

c. How are dx/dt , dy/dt , and dz/dt related if s is constant?

19. Area The area A of a triangle with sides of lengths a and b enclosing an angle of measure θ is

$$A = \frac{1}{2}ab \sin \theta.$$

a. How is dA/dt related to $d\theta/dt$ if a and b are constant?

b. How is dA/dt related to $d\theta/dt$ and da/dt if only b is constant?

c. How is dA/dt related to $d\theta/dt$, da/dt , and db/dt if none of a , b , and θ are constant?

20. Heating a plate When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50 cm?

21. Changing dimensions in a rectangle The length l of a rectangle is decreasing at the rate of 2 cm/sec while the width w is increasing at the rate of 2 cm/sec. When $l = 12$ cm and $w = 5$ cm, find the rates of change of (a) the area, (b) the perimeter, and (c) the lengths of the diagonals of the rectangle. Which of these quantities are decreasing, and which are increasing?

22. Changing dimensions in a rectangular box Suppose that the edge lengths x , y , and z of a closed rectangular box are changing at the following rates:

$$\frac{dx}{dt} = 1 \text{ m/sec}, \quad \frac{dy}{dt} = -2 \text{ m/sec}, \quad \frac{dz}{dt} = 1 \text{ m/sec}.$$

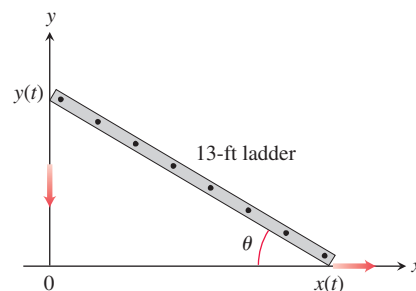
Find the rates at which the box's (a) volume, (b) surface area, and (c) diagonal length $s = \sqrt{x^2 + y^2 + z^2}$ are changing at the instant when $x = 4$, $y = 3$, and $z = 2$.

23. A sliding ladder A 13-ft ladder is leaning against a house when its base starts to slide away (see accompanying figure). By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

a. How fast is the top of the ladder sliding down the wall then?

b. At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?

c. At what rate is the angle θ between the ladder and the ground changing then?



24. Commercial air traffic Two commercial airplanes are flying at an altitude of 40,000 ft along straight-line courses that intersect at right angles. Plane A is approaching the intersection point at a speed of 442 knots (nautical miles per hour; a nautical mile is 2000 yd). Plane B is approaching the intersection at 481 knots. At what rate is the distance between the planes changing when A is 5 nautical miles from the intersection point and B is 12 nautical miles from the intersection point?

25. Flying a kite A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

26. Boring a cylinder The mechanics at Lincoln Automotive are reboring a 6-in.-deep cylinder to fit a new piston. The machine they are using increases the cylinder's radius one-thousandth of an inch every 3 min. How rapidly is the cylinder volume increasing when the bore (diameter) is 3.800 in.?

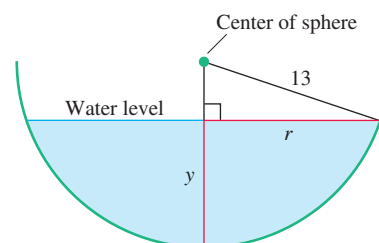
27. A growing sand pile Sand falls from a conveyor belt at the rate of 10 m³/min onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast are the (a) height and (b) radius changing when the pile is 4 m high? Answer in centimeters per minute.

28. A draining conical reservoir Water is flowing at the rate of 50 m³/min from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m.

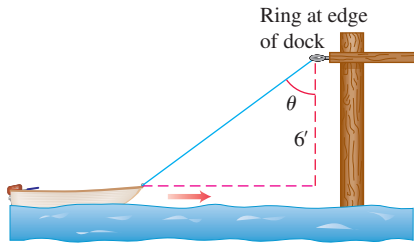
a. How fast (centimeters per minute) is the water level falling when the water is 5 m deep?

b. How fast is the radius of the water's surface changing then? Answer in centimeters per minute.

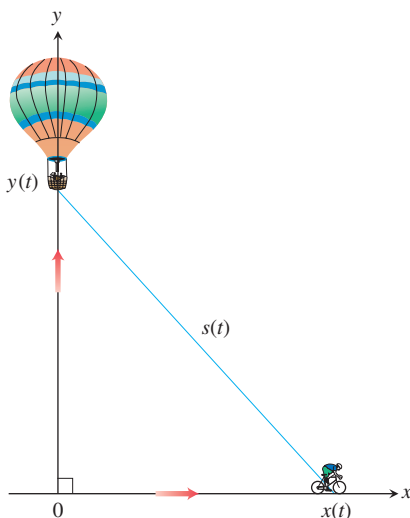
29. A draining hemispherical reservoir Water is flowing at the rate of 6 m³/min from a reservoir shaped like a hemispherical bowl of radius 13 m, shown here in profile. Answer the following questions, given that the volume of water in a hemispherical bowl of radius R is $V = (\pi/3)y^2(3R - y)$ when the water is y meters deep.



- a. At what rate is the water level changing when the water is 8 m deep?
- b. What is the radius r of the water's surface when the water is y m deep?
- c. At what rate is the radius r changing when the water is 8 m deep?
- 30. A growing raindrop** Suppose that a drop of mist is a perfect sphere and that, through condensation, the drop picks up moisture at a rate proportional to its surface area. Show that under these circumstances the drop's radius increases at a constant rate.
- 31. The radius of an inflating balloon** A spherical balloon is inflated with helium at the rate of 100π ft³/min. How fast is the balloon's radius increasing at the instant the radius is 5 ft? How fast is the surface area increasing?
- 32. Hauling in a dinghy** A dinghy is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow. The rope is hauled in at the rate of 2 ft/sec.
- a. How fast is the boat approaching the dock when 10 ft of rope are out?
- b. At what rate is the angle θ changing at this instant (see the figure)?

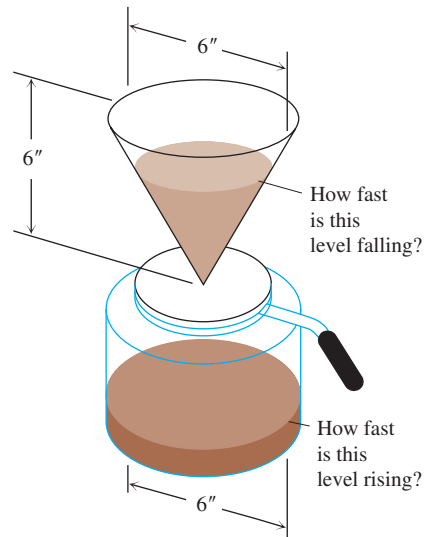


- 33. A balloon and a bicycle** A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance $s(t)$ between the bicycle and balloon increasing 3 sec later?



- 34. Making coffee** Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of 10 in³/min.
- a. How fast is the level in the pot rising when the coffee in the cone is 5 in. deep?

- b. How fast is the level in the cone falling then?



- 35. Cardiac output** In the late 1860s, Adolf Fick, a professor of physiology in the Faculty of Medicine in Würzburg, Germany, developed one of the methods we use today for measuring how much blood your heart pumps in a minute. Your cardiac output as you read this sentence is probably about 7 L/min. At rest it is likely to be a bit under 6 L/min. If you are a trained marathon runner running a marathon, your cardiac output can be as high as 30 L/min.

Your cardiac output can be calculated with the formula

$$y = \frac{Q}{D},$$

where Q is the number of milliliters of CO₂ you exhale in a minute and D is the difference between the CO₂ concentration (ml/L) in the blood pumped to the lungs and the CO₂ concentration in the blood returning from the lungs. With $Q = 233$ ml/min and $D = 97 - 56 = 41$ ml/L,

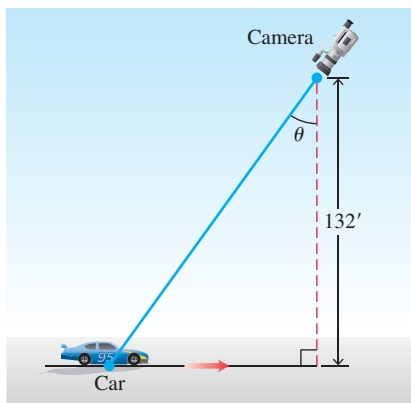
$$y = \frac{233 \text{ ml/min}}{41 \text{ ml/L}} \approx 5.68 \text{ L/min},$$

fairly close to the 6 L/min that most people have at basal (resting) conditions. (Data courtesy of J. Kenneth Herd, M.D., Quillan College of Medicine, East Tennessee State University.)

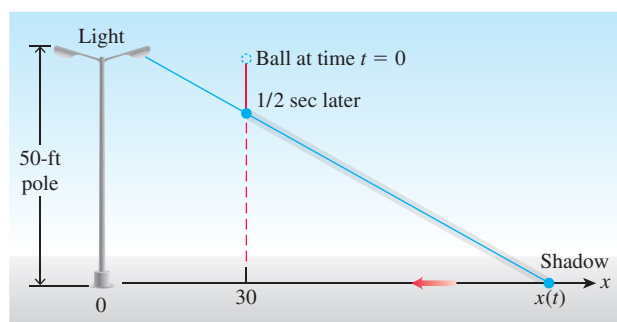
Suppose that when $Q = 233$ and $D = 41$, we also know that D is decreasing at the rate of 2 units a minute but that Q remains unchanged. What is happening to the cardiac output?

- 36. Moving along a parabola** A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x -coordinate (measured in meters) increases at a steady 10 m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = 3$ m?
- 37. Motion in the plane** The coordinates of a particle in the metric xy -plane are differentiable functions of time t with $dx/dt = -1$ m/sec and $dy/dt = -5$ m/sec. How fast is the particle's distance from the origin changing as it passes through the point (5, 12)?
- 38. Videotaping a moving car** You are videotaping a race from a stand 132 ft from the track, following a car that is moving at

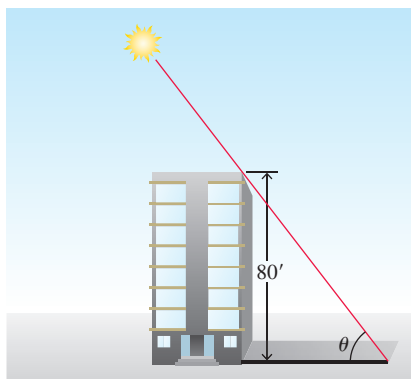
180 mi/h (264 ft/sec), as shown in the accompanying figure. How fast will your camera angle θ be changing when the car is right in front of you? A half second later?



- 39. A moving shadow** A light shines from the top of a pole 50 ft high. A ball is dropped from the same height from a point 30 ft away from the light. (See accompanying figure.) How fast is the shadow of the ball moving along the ground $1/2$ sec later? (Assume the ball falls a distance $s = 16t^2$ ft in t sec.)



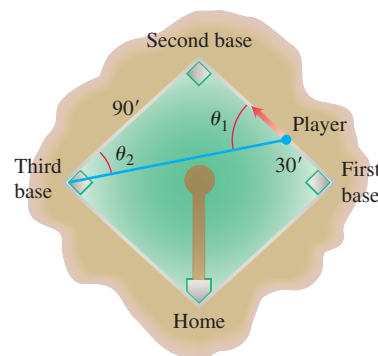
- 40. A building's shadow** On a morning of a day when the sun will pass directly overhead, the shadow of an 80-ft building on level ground is 60 ft long. At the moment in question, the angle θ the sun makes with the ground is increasing at the rate of $0.27^\circ/\text{min}$. At what rate is the shadow decreasing? (Remember to use radians. Express your answer in inches per minute, to the nearest tenth.)



- 41. A melting ice layer** A spherical iron ball 8 in. in diameter is coated with a layer of ice of uniform thickness. If the ice melts at the rate of $10 \text{ in}^3/\text{min}$, how fast is the thickness of the ice decreasing when it is 2 in. thick? How fast is the outer surface area of ice decreasing?

- 42. Highway patrol** A highway patrol plane flies 3 mi above a level, straight road at a steady 120 mi/h. The pilot sees an oncoming car and with radar determines that at the instant the line-of-sight distance from plane to car is 5 mi, the line-of-sight distance is decreasing at the rate of 160 mi/h. Find the car's speed along the highway.

- 43. Baseball players** A baseball diamond is a square 90 ft on a side. A player runs from first base to second at a rate of 16 ft/sec.
- At what rate is the player's distance from third base changing when the player is 30 ft from first base?
 - At what rates are angles θ_1 and θ_2 (see the figure) changing at that time?
 - The player slides into second base at the rate of 15 ft/sec. At what rates are angles θ_1 and θ_2 changing as the player touches base?



- 44. Ships** Two ships are steaming straight away from a point O along routes that make a 120° angle. Ship A moves at 14 knots (nautical miles per hour; a nautical mile is 2000 yd). Ship B moves at 21 knots. How fast are the ships moving apart when $OA = 5$ and $OB = 3$ nautical miles?

- 45. Clock's moving hands** At what rate is the angle between a clock's minute and hour hands changing at 4 o'clock in the afternoon?

- 46. Oil spill** An explosion at an oil rig located in gulf waters causes an elliptical oil slick to spread on the surface from the rig. The slick is a constant 9 in. thick. After several days, when the major axis of the slick is 2 mi long and the minor axis is $3/4$ mi wide, it is determined that its length is increasing at the rate of 30 ft/hr, and its width is increasing at the rate of 10 ft/hr. At what rate (in cubic feet per hour) is oil flowing from the site of the rig at that time?

- 47. A lighthouse beam** A lighthouse sits 1 km offshore, and its beam of light rotates counterclockwise at the constant rate of 3 full circles per minute. At what rate is the image of the beam moving down the shoreline when the image is 1 km from the spot on the shoreline nearest the lighthouse?

