Math 21A, Winter 2023.

Discussion Problems 1 (Tue., Jan. 23)

1. Compute the following limits:

(a) $\lim\limits_{x\to 2} \dfrac{x^2-4}{x-2}$ (b) $\lim\limits_{x\to 1} \dfrac{x-1}{2-\sqrt{x+3}}$ (c) $\lim\limits_{x\to 8} \dfrac{\sqrt{2x}-4}{\sqrt{x+1}-3}$

(d) $\lim\limits_{x\to 0} x\cdot\cos(x^3+17x^{-3})$ (e) $\lim\limits_{x\to 1} x\cdot\cos(x^3+17x^{-3})$

2. Let

$$f(x) = \begin{cases} x^2+a & x\le 1 \\ 1-x & x>1 \end{cases}$$

(a) Determine a so that $\lim\limits_{x\to 1} f(x)$ exists.

(b) For a as determined in (a), is the function $y=f(x)$ continuous for all x?

(c) For a as determined in (a), graph the function $y=f(x)$ and determine its range.

3. Let

$$f(x) = \begin{cases} x^2 & x\le 1 \\ Ax+B & 1<x<2 \\ -2x^2 & 2\le x \end{cases}$$

(a) Determine the numbers A and B so that $y=f(x)$ is continuous everywhere.

(b) Then graph the function $y=f(x)$ and determine its range.

4. Compute the following limits:

(a) $\lim\limits_{x\to 0} \dfrac{\sin 7x}{\sin 3x}$ (b) $\lim\limits_{x\to \pi} \dfrac{\sin 7x}{\sin 3x}$ (c) $\lim\limits_{x\to \infty} \dfrac{\sin 7x}{3x}$