Math 21A, Winter 2024.

**Discussion Problems 4 (Tue., Feb. 13)**

1. Let $y = \sin(2x) + x^{517}$. Compute $y^{(1000)}$, the 1000th derivative of $y$.

2. Is there a number $x$ in $(0, \pi/2)$ at which the tangents to $y = \sin(2x)$ and $y = \tan x$ have the same slope?

3. Let $f(x) = \sin^3(x^2)$. Find the equation of the tangent line to $y = f(x)$ at $x = \sqrt{\pi/4}$.

4. Assume $y = f(x)$ and $y = g(x)$ are differentiable functions, $f(3) = 2$, $f'(3) = 3$, $g(2) = 3$, $g'(2) = 5$, $f(3) = 2$. Let $h(x) = f(g(x))$ and $k(x) = g(f(x))$. Find $h'(2)$ and $k'(3)$.

5. A function $y = f(x)$ satisfies

$$(x - y)^3 = x^2 - y^2 - 2$$

Find the equation of the tangent to the graph of this function at the point $(2, 1)$. At which point does the tangent cross the $x$-axis and at what angle?

6. A function $y = f(x)$ satisfies

$$xy = y^2 - 1$$

Determine the first derivative $y'$ and the second derivative $y''$ of this function at the point $(0, 1)$.