Math 21A, Winter 2024.

Discussion Problems 6 (Tue., Feb. 27)

1. Find all critical points of $y = f(x)$ given by

$$f(x) = x + 5\arctan\frac{1}{x}$$

on the interval $(0, \infty)$. Then find the global maximum and minimum of $y = f(x)$ on the interval $[1, 5]$.

2. Find domain and range of the function f given by $f(x) = \sqrt{x^2 - x^4}$.

3. A particle is moving on a coordinate line. Its position at time $t \geq 0$ is given by $s = f(t)$, where $f(t) = 4t - \cos(2t)$. How many times does the particle visit the origin (i.e., its position is $s = 0$)?

4. Let $f(x) = x^2(x-4)^{2/3}$. Find the global maximum and the global minimum of $y = f(x)$ on $[0, 5]$ and on $[-4, 4]$.

5. Assume that a function f is defined, continuous and differentiable for all x. Give a *precise* argument for your answer on the following two questions.
(a) If $f(1) = 1$ and $f(2) = 3$, is it possible that $f'(x) > 3$ for all x?
(b) If $f(0) = 5$ and $f'(0) = -1$, is it possible that $f(x) \leq 5$ for all x?

6. Consider the function $f(x) = x^4 - 2x^2$ on the domain $D = [0, 2]$.
(a) Find the range of $y = f(x)$ on D.
(b) Find the range of $y = f(x)^2$ on D.
(c) Find the range of $y = \cos\left(\frac{\pi}{4} f(x)\right)$ on D.
(d) Find the range of $y = \sin\left(\frac{\pi}{6} f(x)\right)$ on D.