Discussion Problems 8 (Tue., Mar. 12)

These problems are good preparation for the final exam.

1. Find the point on the graph of $y = \sqrt{x}$ which is nearest to the point $(4, 0)$.

2. Two posts, one 12 feet hight and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires, attached to a single stake on the ground and running to the top of each post.
(a) Where should the stake be placed so that the least amount of wire is used?
(b) Can you solve the problem without calculus? (*Hint*: reflect one of the posts.)
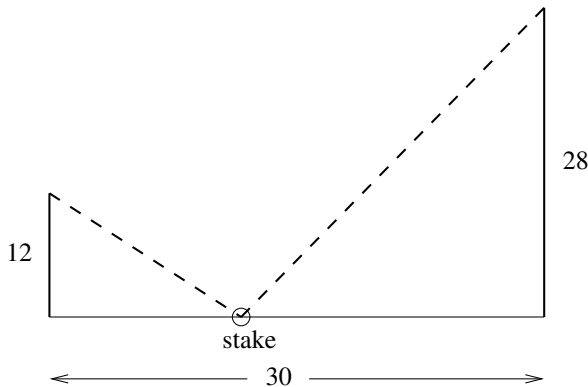
3. (This is Problem 53(a,b,c,e) in Section 4.6 of the book, on distance between two ships.) At noon, ship A was 12 nautical miles due north of ship B. Ship A was sailing south at 12 knots (nautical miles per hour) and continued to do so all day. Ship B was sailing east at 8 knots and continued to do so all day.
(a) Start counting time with $t = 0$ at noon and express the distance s between the ships as a function of t.
(b) How rapidly was the distance between the ships changing at noon? One hour later?
(c) The visibility that day was 5 nautical miles. Did the ships ever sight each other?
(d) Compute $\lim_{t \to \infty} \dfrac{ds}{dt}$.

4. Compute the following limits:

(a) $\lim_{x \to 0} \dfrac{1 - \cos(2x)}{xe^x - x}$

(b) $\lim_{x \to 0} (1 + 2x)^{5/x}$

(c) $\lim_{x \to 0} \left(\dfrac{1}{x} - \dfrac{2}{\sin(2x)} \right)$

(d) $\lim_{x \to 0+} (e^x - 1) \ln x$

(e) $\lim_{x \to \infty} (3^x + 4^x)^{1/x}$

(f) $\lim_{x \to 0} \dfrac{e^{-1/x^2}}{x}$