MAT 21A-C, Fall 2005 (J. Gravner)

SAMPLE EXAMS

Note. These exams were given a few years ago, when a different book was used, and the syllabus was a little different. They are provided to give you the basic idea about the length and difficulty of the exams in this class. For practice, solve the problems *before looking at the solutions*!

MIDTERM EXAM 1

1. Let $f(x) = \frac{x^2-x-6}{x^2-1}$. Compute vertical and horizontal asymptotes, ans sketch the graph of this function. On your graph, identify clearly all the asymptotes and intercepts.

2. Compute the following limits. In each of the four cases, you should state your result as a finite number, $+\infty$ or $-\infty$.

(a) $\lim_{x\to\infty} \dfrac{x^2+3x+\sin x}{3(x+2)^2}$.

(b) $\lim_{x\to 0} \dfrac{(1+\cos(5x))\sin(7x)}{\sin(9x)}$.

(c) $\lim_{x\to 5} \dfrac{\sqrt{x-1}-2}{\sqrt{x+4}-3}$.

(d) $\lim_{x\to 4+} \dfrac{\sin x}{4-x}$.

3. Below are 3 questions. Provide straightforward answers with clear explanations.

(a) Can a be chosen so that

$$f(x) = \begin{cases} \frac{|x|(x^2-4)}{x+2}, & \text{if } x \neq -2, \\ a, & \text{if } x = -2 \end{cases}$$

is a continuous function at every x?

(b) Does the equation $x = 10\sin x$ have a solution for $x > 0$? (Note that the solution $x = 0$ does *not* count!)

(c) Is the function $f(x) = x + |x|$ differentiable at 0?

4. Find the points on the graph of the function

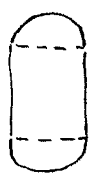
$$y = \frac{x+1}{x-1}$$

at which the tangent line goes through the point (0,0).

MIDTERM EXAM 2

1.
(a) $f(x) = \dfrac{x+2}{\sqrt{x^2+x+1}+1}$ Compute $f'(x)$. *Do not simplify.*
(b) $f(x) = \sqrt{1+\sqrt{\cos x}}$. Compute $f'(x)$. *Do not simplify.*
(c) Assume that $y = f(x)$ satisfies the equation $x^3 + xy^2 + y^3 = 3$. Compute the equation of the tangent line to the graph of $y = f(x)$ at the point $(1, 1)$.
2. Graph the function $f(x) = 4\sqrt[3]{x} + x\sqrt[3]{x}$ using the first and second derivative. Determine all relative extrema and inflection points. Clearly label all important points on the graph. Determine the range of this function.

(A little help to check your computations. Two of the following 3 expressions are $f'(x)$ and $f''(x)$: $\frac{4}{9}x^{-5/3}(x-2)$, $\frac{9}{4}x^{-1/3}(x-7)$, $\frac{4}{3}x^{-2/3}(1+x)$.)
3. Below are 3 questions. Provide straightforward answers with clear explanations. In each question, f is assumed to be a continuous function, defined for all x, and such that $f'(x)$ and $f''(x)$ exist for all x.
(a) $f(1) = 5$, $f(3) = 6$. Is it possible that $f'(x) > 1$ for all x?
(b) $f'(x) = \dfrac{x}{1+x^2}$. How many inflection points does $y = f(x)$ have?
(c) $f(0) = 5$, $f(9) = 6$, $f''(x) = \dfrac{-x}{17+x^2+x^4}$. Is it possible that $f(4) = 2$?
4. A racetrack field consists of a rectangular piece and two semi-circular pieces at the end. (See the picture.) The actual racetrack is build along the perimeter of the field. Assume that the area of the field is constrained to be 1000 square feet. What is the length of the shortest possible racetrack?



FINAL EXAM

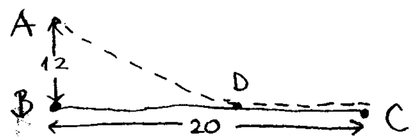
1. Compute the following limits, *in any correct way* you can.
(a) $\displaystyle\lim_{x\to\infty} \frac{x^4 + x^2\cos x + 1}{5x^4 - 1000x^3 - 1}$
(b) $\displaystyle\lim_{h\to 0} \frac{\tan(\frac{\pi}{4}+h) - 1}{h}$
(c) $\displaystyle\lim_{x\to 0} \frac{e^{2x} - 2x - 1}{(\tan x)^2}$
(d) $\displaystyle\lim_{x\to\infty} \left(\frac{3x-1}{3x+2}\right)^{3x+4}$
2.
(a) $f(x) = \ln(1 + e^{\sqrt{x}})$. Compute $f'(x)$. *Do not simplify.*
(b) $f(x) = \arcsin(x \cdot \ln x)$. Compute $f'(x)$. *Do not simplify.*
(c) Assume that $f(x)$ satisfies the equation $x \cdot f(x)^5 + \tan\left(\frac{\pi}{4} f(x)\right) = 2x$ and that $f(1) = 1$. Compute $f'(1)$.

3.
(a) Let $f(x) = \frac{x}{x^2 - 16}$. Verify that $\lim_{x \to \infty} f(x) = 0$. Then find an *integer* M which guarantees that $f(x) < 0.001$ if $x \geq M$.
(b) You are standing on top of a 32 ft tall tower. You throw a rock straight *down* with velocity 16 ft/sec. How fast will the rock travel when it hits the ground? Assume the accelleration of the rock is constantly -32ft/sec^2.

4. Graph the function $f(x) = \frac{\ln x}{x^2}$ using the first and second derivative. Be sure to label clearly all important points on the graph.

What is the range of the function? Is the fuction f one–to–one on the interval $(1, 2)$? Explain.

5. You are sitting in a truck on a flat prairie at the point A (see the picture) 12 miles from a straight paved road. You want to reach the point C, 20 miles from the closest point B on the road. Your plan is to drive in a straight line to a point D on the road, then proceed to drive on the road until you reach C (dashed line in the picture). You can drive twice as fast on the road as you can on the prairie. Locate the point D which minimizes your driving time from A to C.



6. Provide straightforward, and fully justified, answers to the following questions. In each of them, assume that $f(x)$ (if not explicitly given) is a continuous function defined for all x, and $f'(x)$ and $f''(x)$ are continuous for all x.
(a) Is it possible that $f'(x) < 0$ for all x, yet $y = f(x)$ has no x intercepts?
(b) Let $f(x) = \sin(3x) - 5x + 2$. How many x intercepts does $y = f(x)$ have?
(c) $f''(x) = \frac{x^2(1 - x^2)}{1 + x^4}$. How many inflection points does f have?
(d) $f'(0) = f'(1) = 0$ and $f'(x) \neq 0$ at all x other than $0, 1$. Is it possible that $y = f(x)$ has local maxima at both $x = 0$ and $x = 1$?

7. Recall that the first quadrant in the coordinate plane consists of points (x, y) such that $x \geq 0$ and $y \geq 0$.
(a) At which points on the curve $y = e^{-x}$ does the tangent intersect the first quadrant?
(b) What is the largest possible area of the triangle formed in the first quadrant by the x-axis, y-axis and a tangent to $y = e^{-x}$? What is the largest possible *perimeter* of such triangle?

MT 1 KEY

1. Let $f(x) = \frac{x^2-x-6}{x^2-1}$. Compute vertical and horizontal asymptotes, ans sketch the graph of this function. On your graph, identify clearly all the asymptotes and intercepts.

$$f(x) = \frac{(x-3)(x+2)}{(x-1)(x+1)}$$

Domain: $x \neq 1, x \neq -1$.

Vertical asymptotes

$$\lim_{x \to 1^+} f(x) = -\infty$$

$$\lim_{x \to 1^-} f(x) = +\infty$$

$$\lim_{x \to -1^+} f(x) = +\infty$$

$$\lim_{x \to -1^-} f(x) = -\infty$$

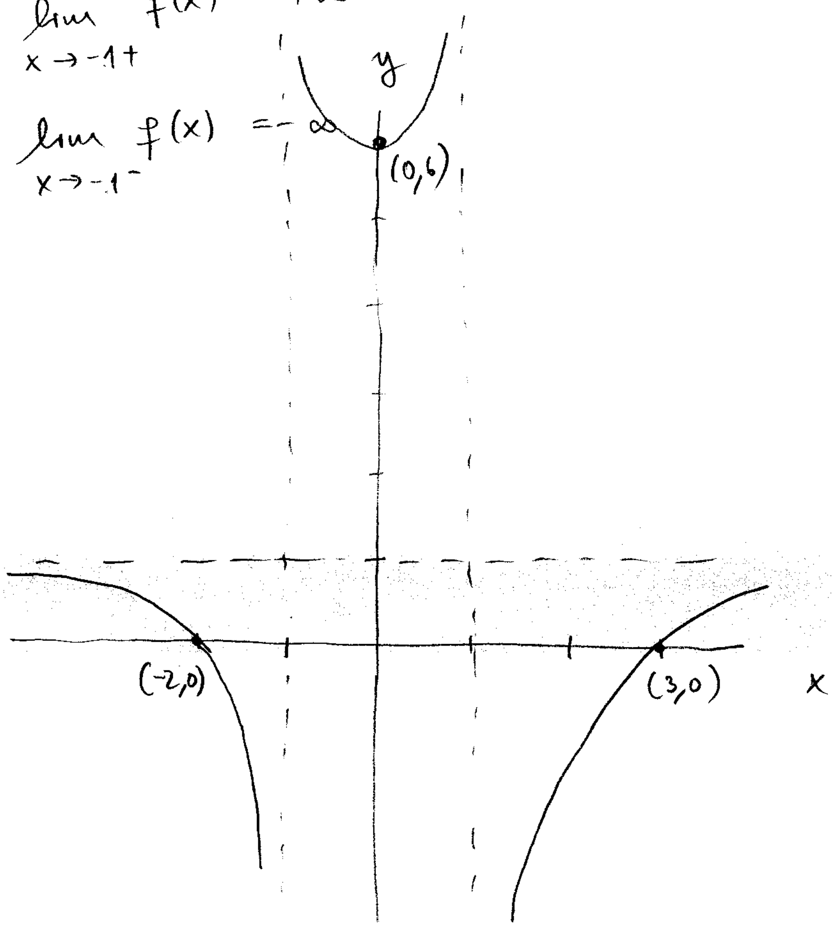
Intercepts: $(0,6)$

$(3,0)$, $(-2,0)$

Horizontal asymptote

$$\lim_{x \to \infty} f(x) = 1$$

$$\lim_{x \to -\infty} f(x) = 1$$



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2. Compute the following limits. In each of the four cases, you should state your result as a finite number, $+\infty$ or $-\infty$.

(a) $\lim_{x\to\infty} \frac{x^2+3x+\sin x}{3(x+2)^2}$. $= \boxed{\frac{1}{3}}$

(b) $\lim_{x\to 0} \frac{(1+\cos(5x))\sin(7x)}{\sin(9x)}$. $= \lim_{x\to 0} \underset{\substack{\downarrow \\ 2}}{(1+\cos 5x)} \cdot \underset{\substack{\downarrow \\ 1}}{\frac{\sin 7x}{7x}} \cdot \underset{\substack{\downarrow \\ 1}}{\frac{9x}{\sin 9x}} \cdot \frac{7}{9} = \boxed{\frac{14}{9}}$

(c) $\lim_{x\to 5} \frac{\sqrt{x-1}-2}{\sqrt{x+4}-3}$. $= \lim_{x\to 5} \frac{x-1-4}{x+4-9} \cdot \frac{\sqrt{x+4}+3}{\sqrt{x-1}+2} = \frac{6}{4} = \boxed{\frac{3}{2}}$

(d) $\lim_{x\to 4^+} \frac{\sin x}{4-x}$. $= +\infty$

(since $\sin 4 < 0$)

3. Below are 3 questions. Provide straightforward answers with clear explanations. ~~In each question, f is assumed to be a continuous function, defined for all x, and such that $f'(x)$ and $f''(x)$ exist for all x.~~

(a) Can a be chosen so that

$$f(x) = \begin{cases} \cancel{\frac{|x|(x+2)}{x^2-4}} \; \frac{|x|(x^2-4)}{x+2}, & \text{if } x \neq -2, \\ a, & \text{if } x = -2 \end{cases}$$

is a continuous function at every x?

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} |x|(x-2) = 2(-4) = -8$$

Yes. $\boxed{a = -8}$

(b) Does the equation $x = 10 \sin x$ have a solution for $x > 0$? (Note that the solution $x = 0$ does *not* count!)

$$f(x) = x - 10 \sin x$$

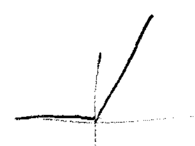
$$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} - 10 < 0$$

$$f(11) = 11 - 10 \sin 11 \geq 1 > 0$$

Yes. By IVT, there is an $x \in \left(\frac{\pi}{2}, 10\right)$ such that $f(x) = 0$.

(c) Is the function $f(x) = x + |x|$ differentiable at 0?

$$f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



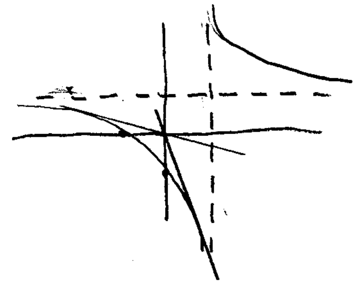
No: $$\lim_{h \to 0^+} \frac{f(h) - f(0)}{h} = 2$$

$$\lim_{h \to 0^-} \frac{f(h) - f(0)}{h} = 0$$

4. Find the points on the graph of the function

$$y = \frac{x+1}{x-1}$$

at which the tangent line goes through the point (0,0).



$$y' = \frac{x-1 - (x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

Pt. $\left(a, \frac{a+1}{a-1}\right)$

Tangent $\quad y = -\frac{2}{(a-1)^2}(x-a) + \frac{a+1}{a-1}$

Goes through $(0,0)$:

$$\frac{2a}{(a-1)^2} = \frac{a+1}{a-1}$$

$$\frac{2a}{a-1} = a+1$$

$$a^2 + 2a - 1 = 0$$

$$a = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$$

Two points:

$$\left(-1-\sqrt{2}, \frac{-\sqrt{2}}{-2-\sqrt{2}}\right) = \boxed{\left(-1-\sqrt{2}, \sqrt{2}-1\right)}$$

$$\left(-1+\sqrt{2}, \frac{\sqrt{2}}{-2+\sqrt{2}}\right) = \boxed{\left(-1+\sqrt{2}, -1-\sqrt{2}\right)}$$

MT2 KEY

1.

(a) $f(x) = \dfrac{x+2}{\sqrt{x^2+x+1}+1}$ Compute $f'(x)$. *Do not simplify.*

$$f'(x) = \frac{\sqrt{x^2+x+1} + 1 - (x+2) \cdot \frac{1}{2\sqrt{x^2+x+1}}(2x+1)}{\left(\sqrt{x^2+x+1} + 1\right)^2}$$

(b) $f(x) = \sqrt{1+\sqrt{\cos x}}$. Compute $f'(x)$. *Do not simplify.*

$$f'(x) = \frac{1}{2\sqrt{1+\sqrt{\cos x}}} \cdot \frac{1}{2\sqrt{\cos x}} \cdot (-\sin x)$$

(c) Assume that $y = f(x)$ satisfies the equation $x^3 + xy^2 + y^3 = 3$. Compute the equation of the tangent line to the graph of $y = f(x)$ at the point $(1, 1)$.

$$3x^2 + y^2 + 2xyy' + 3y^2 \cdot y' = 0 \qquad \text{at } (1,1):$$

$$y' = -\frac{4}{5}$$

tangent line: $y - 1 = -\frac{4}{5}(x-1)$

$$f(x) = 4x^{1/3} + x^{4/3} = x^{1/3}(4+x)$$

2. Graph the function $f(x) = 4\sqrt[3]{x} + x\sqrt[3]{x}$ using the first and second derivative. Determine all relative extrema and inflection points. Clearly label all important points on the graph. Determine the range of this function.

(A little help to check your computations. Two of the following 3 expressions are $f'(x)$ and $f''(x)$: $\frac{4}{9}x^{-5/3}(x-2)$, $\frac{9}{4}x^{-1/3}(x-7)$, $\frac{4}{3}x^{-2/3}(1+x)$.)

$$f'(x) = \frac{4}{3}x^{-2/3} + \frac{4}{3}x^{1/3} = \frac{4}{3}x^{-2/3}(1+x)$$

$$f''(x) = \frac{4}{3}\left(-\frac{2}{3}\right)x^{-5/3} + \frac{4}{3}\cdot\frac{1}{3}x^{-2/3} = \frac{4}{9}x^{-5/3}(-2+x)$$

	$(-\infty,-1)$	$(-1,0)$	$(0,\infty)$
sign of f'	$-$	$+$	$+$

relative min at $(-1,-3)$

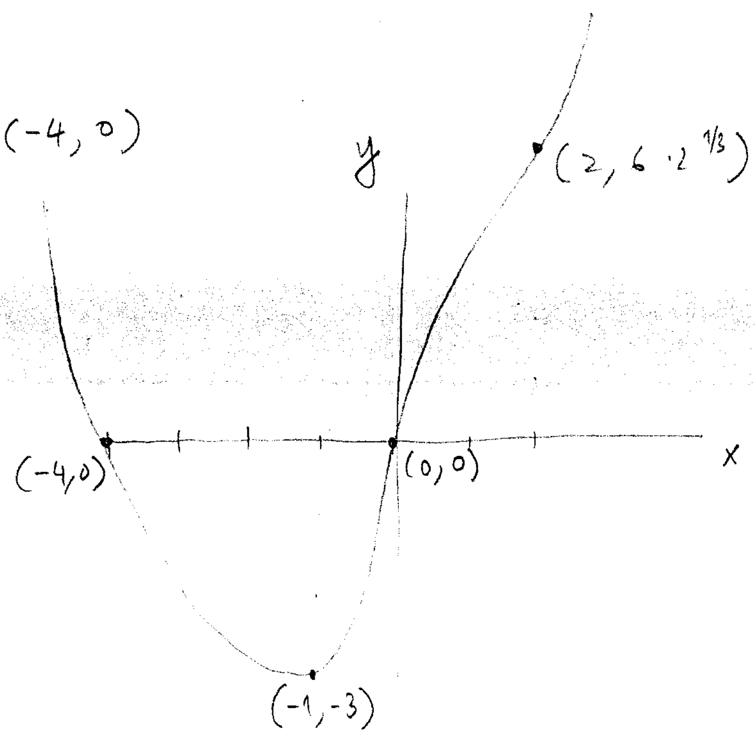
	$(\infty,0)$	$(0,2)$	$(2,\infty)$
sign of f''	$+$	$-$	$+$

two inflection points: $(0,0)$, $(2, 6\cdot 2^{1/3})$

domain: all x

intercept: $(0,0)$, $(-4,0)$

range: $[-3,\infty)$



3. Below are 3 questions. Provide straightforward answers with clear explanations. In each question, f is assumed to be a continuous function, defined for all x, and such that $f'(x)$ and $f''(x)$ exist for all x.

(a) $f(1) = 5$, $f(3) = 6$. Is it possible that $f'(x) > 1$ for all x?

$\boxed{\text{No.}}$ $f'(c) = \dfrac{f(3) - f(1)}{3-1} = \dfrac{1}{2}$ for some c in $(1,3)$

(b) $f'(x) = \dfrac{x}{1+x^2}$. How many inflection points does $y = f(x)$ have?

$$f''(x) = \frac{1+x^2 - 2x \cdot x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$f''(x) = 0$ at $x = -1, 1$

	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
sign of f''	$-$	$+$	$-$

$\boxed{2}$ inflection points.

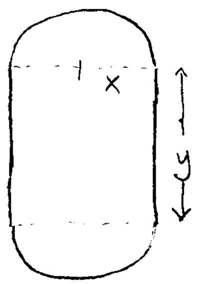
(c) $f(0) = 5$, $f(9) = 6$, $f''(x) = \dfrac{-x}{17 + x^2 + x^4}$. Is it possible that $f(4) = 2$?

$\boxed{\text{No.}}$ f is a concave down function on $(0,9)$

Any critical pt. in $(0,9)$ is then a local max, so the minimum of f must be achieved at an endpoint.

So $f(x) \geq 5$ on $(0,9)$.

4. A racetrack field consists of a rectangular piece and two semi–circular pieces at the end. (See the picture.) The actual racetrack is build along the perimeter of the field. Assume that the area of the field is constrained to be 1000 square feet. What is the length of the shortest possible racetrack?



$$\boxed{\begin{aligned} &\pi x^2 + 2xy = 1000 \\ &\text{minimize: } 2\pi x + 2y \end{aligned}} \qquad y = \frac{1000 - \pi x^2}{2x}$$

$$f(x) = 2\pi x + \frac{1000 - \pi x^2}{x}.$$

$$= \pi x + \frac{1000}{x} \qquad x \geq 0$$

$$f'(x) = \pi - \frac{1000}{x^2}$$

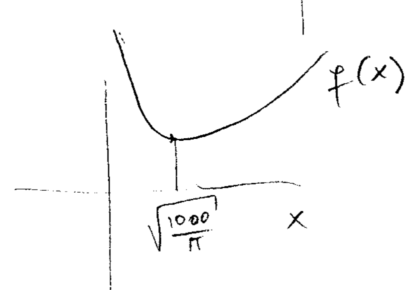
$$f'(x) = 0 \text{ when } x = \sqrt{\frac{1000}{\pi}}$$

At $x = \sqrt{\frac{1000}{\pi}}$:

$$\boxed{f(x) = 2\sqrt{1000\pi}}$$

Why is this the minimum

	$\left(0, \sqrt{\frac{1000}{\pi}}\right)$	$\left(\sqrt{\frac{1000}{\pi}}, \infty\right)$
sign of f'	$-$	$+$



$\boxed{\text{FINAL KEY}}$ T

1. Compute the following limits, *in any correct way* you can.

(a) $\lim_{x\to\infty} \frac{x^4 + x^2\cos x + 1}{5x^4 - 1000x^3 - 1}$ $= \boxed{\frac{1}{5}}$ (divide top & bottom by x^4)

(b) $\lim_{h\to 0} \frac{\tan(\frac{\pi}{4}+h) - 1}{h}$ $= f'(\frac{\pi}{4}) = \boxed{2}$ where $f(x) = \tan x$

$f'(x) = \frac{1}{\cos^2 x}$, at $x = \frac{\pi}{4}$ equals 2.

(c) $\lim_{x\to 0} \frac{e^{2x} - 2x - 1}{(\tan x)^2}$ $= \lim_{x\to 0} \frac{e^{2x} - 2x - 1}{\sin^2 x} = \lim_{x\to 0} \frac{2e^{2x} - 2}{2\sin x \cancel{\cos x}}$

($\cos x \to 1$) $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{4e^{2x}}{2\cos x} = \boxed{2}$

(d) $\lim_{x\to\infty} \left(\frac{3x-1}{3x+2}\right)^{3x+4}$ $= \lim_{x\to\infty} \left(1 - \frac{3}{3x+2}\right)^{-\frac{3x+2}{3} \cdot \frac{-(3x+4)\cdot 3}{3x+2}}$

$= \boxed{e^{-3}}$

2.

(a) $f(x) = \ln(1 + e^{\sqrt{x}})$. Compute $f'(x)$. *Do not simplify.*

$$f'(x) = \frac{1}{1 + e^{\sqrt{x}}} \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

(b) $f(x) = \arcsin(x \cdot \ln x)$. Compute $f'(x)$. *Do not simplify.*

$$f'(x) = \frac{1}{\sqrt{1 - (x \ln x)^2}} (1 + \ln x)$$

(c) Assume that $f(x)$ satisfies the equation $x \cdot f(x)^5 + \tan\left(\frac{\pi}{4} f(x)\right) = 2x$ and that $f(1) = 1$. Compute $f'(1)$.

$$f(x)^5 + x \cdot 5 f(x)^4 \cdot f'(x) + \frac{1}{\cos^2(\frac{\pi}{4} f(x))} \cdot \frac{\pi}{4} \cdot f'(x) = 2$$

$$x = 1, \; f(x) = 1$$

$$1 + 5 f'(x) + 2 \cdot \frac{\pi}{4} \cdot f'(x) = 2$$

$$f'(x) = \boxed{\frac{1}{5 + \frac{\pi}{2}}}$$

T

3.
(a) Let $f(x) = \frac{x}{x^2 - 16}$. Verify that $\lim_{x \to \infty} f(x) = 0$. Then find an *integer* M which guarantees that $f(x) < 0.001$ if $x \geq M$.

$$f(x) = \frac{1}{x - \frac{16}{x}}$$ when x is large, $\frac{16}{x} \approx 0$.

If $x > 16$, $\frac{16}{x} < 1$, so that

$$f(x) > \frac{1}{x-1}.$$

And $\frac{1}{x-1} < \frac{1}{1000}$ if $x > 1001$. Take

$\boxed{M = 1001}$.

(b) You are standing on top of a 32 ft tall tower. You throw a rock straight *down* with velocity 16 ft/sec. How fast will the rock travel when it hits the ground? Assume the accelleration of the rock is constantly -32ft/sec^2.

$$f(t) = -16t^2 - \cancel{+}\, 16t + 32$$
$$= -16\,(t^2 + t - 2) = -16\,(t-1)(t+2)$$

$t = 1$, $f'(1) = -32t - 16 \,\Big|_{t=1}$
$$= \underline{\underline{-48}} \text{ ft/sec}$$

J

4. Graph the function $f(x) = \dfrac{\ln x}{x^2}$ using the first and second derivative. Be sure to label clearly all important points on the graph.

What is the range of the function? Is the fuction f one–to–one on the interval $(1,2)$? Explain.

$$f'(x) = \frac{\frac{1}{x}\cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{1-2\ln x}{x^3}$$

$$f''(x) = \frac{-\frac{2}{x}\cdot x^3 - (1-2\ln x)\cdot 3x^2}{x^6} = \frac{-5+6\ln x}{x^4}$$

$$\boxed{\lim_{x\to\infty} \frac{\ln x}{x^2} = 0 \qquad \lim_{x\to 0^+} \frac{\ln x}{x^2} = -\infty}$$

Domain: $x > 0$

Intercept: $(1,0)$

$f'(x) = 0$ when $\ln x = \frac{1}{2}$, $x = e^{1/2}$

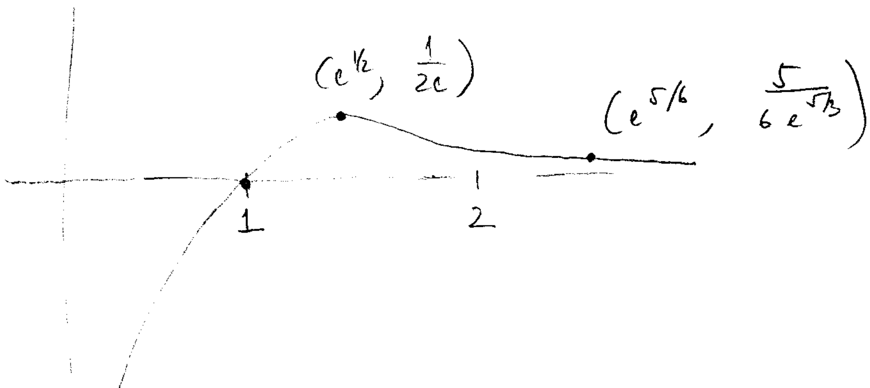
interval	$(0, e^{1/2})$	$(e^{1/2}, \infty)$
sign of f'	+	−

$\left(e^{1/2}, \frac{1}{2e}\right)$ local max

$f''(x) = 0$ when $\ln x = \frac{5}{6}$, $x = e^{5/6}$

interval	$(0, e^{5/6})$	$(e^{5/6}, \infty)$
sign of f''	−	+

$\left(e^{5/6}, \frac{5}{6e^{5/3}}\right)$ infl. pt.

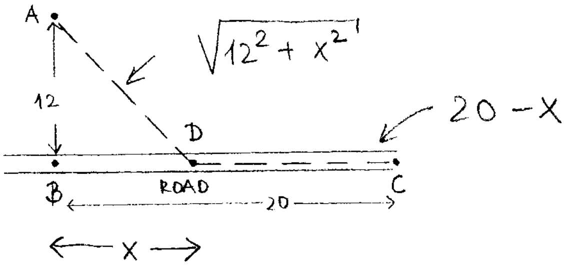


Range: $\left(-\infty, \frac{1}{2e}\right]$

Not one-to-one on $(1,2)$, because this interval includes the local max.

J

5. You are sitting in a truck on a flat prairie at the point A (see the picture) 12 miles from a straight paved road. You want to reach the point C, 20 miles from the closest point B on the road. Your plan is to drive in a straight line to a point D on the road, then proceed to drive on the road until you reach C (dashed line in the picture). You can drive twice as fast on the road as you can on the prairie. Locate the point D which minimizes your driving time from A to C.



$$f(x) = 2\sqrt{12^2+x^2} + 20 - x \qquad 0 \le x \le 20$$

$$f'(x) = 2\frac{x}{\sqrt{12^2+x^2}} - 1 = 0$$

$$4x^2 = 12^2 + x^2$$

$$3x^2 = 12^2$$

$$x^2 = 3 \cdot 16$$

$\sqrt{12^2+x^2} = 8\sqrt{3}$ $\qquad x = \boxed{4\sqrt{3}}$ (the only positive critical number)

$$f''(x) = 2\,\frac{\sqrt{12^2+x^2} - \frac{x}{2\sqrt{12^2+x^2}}}{12^2+x^2} = 2\,\frac{8\sqrt{3} - \frac{4\sqrt{3}}{16\sqrt{3}}}{\cdots} > 0,$$

JF $\underline{\text{minimum}}$,

6. Provide straightforward, and fully justified, answers to the following questions. In each of them, assume that $f(x)$ (if not explicitly given) is a continuous function defined for all x, and $f'(x)$ and $f''(x)$ are continuous for all x.

J (a) Is it possible that $f'(x) < 0$ for all x, yet $y = f(x)$ has no x intercepts?

$\boxed{\text{Yes}}$ $f(x) = e^{-x}$, for example. ? for $-x^2-1$

J (b) Let $f(x) = \sin(3x) - 5x + 2$. How many x intercepts does $y = f(x)$ have?

$\boxed{1}$ $f'(x) = 3\cos(3x) - 5 \leq -2$ so $f'(x) \neq 0$ ever, so at most one.

$\left.\begin{array}{l} \lim_{x\to\infty} f(x) = -\infty \\ \lim_{x\to-\infty} f(x) = \infty \end{array}\right\}$ so at least one, by IVP.

P (c) $f''(x) = \dfrac{x^2(1-x^2)}{1+x^4}$. How many inflection points does f have?

$\boxed{2}$

$f''(x) = 0$ at $0, 1, -1$

	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
sign of f''	−	+	+	−

P (d) $f'(0) = f'(1) = 0$ and $f'(x) \neq 0$ at all x other than $0, 1$. Is it possible that $y = f(x)$ has local maxima at both $x = 0$ and $x = 1$?

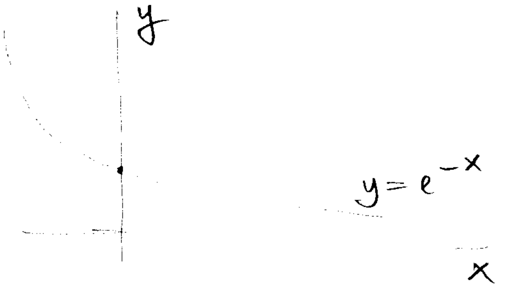
$\boxed{\text{No}}$ If f has a local max. at 0, then:

	$(-\infty,0)$	$(0,1)$	$(1,\infty)$
sign of f'	−	+	?

can't be − + !

P

7. Recall that the first quadrant in the coordinate plane consists of points (x, y) such that $x \geq 0$ and $y \geq 0$.
(a) At which points on the curve $y = e^{-x}$ does the tangent intersect the first quadrant?



Tangent at (a, e^{-a}):

$$y - e^{-a} = -e^{-a}(x-a)$$

$$y = -e^{-a}x + e^{-a}(a+1)$$

When $\boxed{a \geq -1}$.

(b) What is the largest possible area of the triangle formed in the first quadrant by the x-axis, y-axis and a tangent to $y = e^{-x}$? What is the largest possible *perimeter* of such triangle?

y

Perimeter could be <u>arbitrarily large</u>.

$\cdot (0, e^{-a}(a+1))$

$(a+1, 0)$ x

$$f(a) = e^{-a}(a+1)^2 \qquad a \in [-1, \infty),$$

$$f'(a) = -e^{-a}(a+1)^2 + 2(a+1)e^{-a}$$

$$= (a+1)e^{-a}(2 - a - 1) = 0$$

when $a = 1$

$\boxed{f(1) = 4e^{-1}}$ ← largest area

Must be max, because $f(-1) = 0$

$\lim_{a \to \infty} f(a) = 0$,