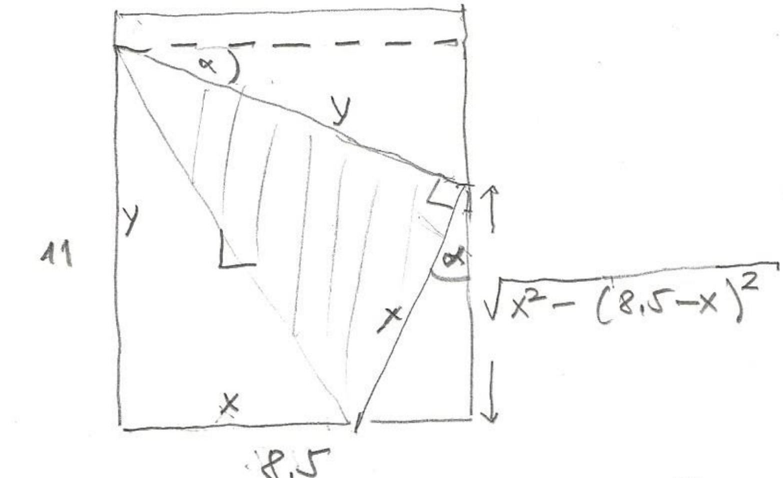
4.6. Problem 25



We need to minimize the length L of the crease.

$$L^2 = x^2 + y^2$$

Similar triangles: $\dfrac{y}{8.5} = \dfrac{x}{\sqrt{x^2-(8.5-x)^2}}$

$$y^2 = \frac{(8.5)^2 x^2}{x^2-(8.5-x)^2} = \frac{(8.5)^2 x^2}{2\cdot 8.5x - (8.5)^2} = \frac{8.5x^2}{2x-8.5}$$

$$L^2 = x^2 + \frac{8.5x^2}{2x-8.5} = \frac{2x^3 - 8.5x^2 + 8.5x^2}{2x-8.5}$$

$$= \frac{2x^3}{2x-8.5} = f(x)$$

We need to minimize $f(x)$ for x in $\left[\frac{8.5}{2}, 8.5\right] = [4.25, 8.5]$

$$f'(x) = 2\,\frac{3x^2(2x-8.5) - 2x^3}{(2x-8.5)^2}$$

$$= 2x^2 \cdot \frac{4x - 25.5}{(2x-8.5)^2} \qquad \text{C.P.: } x = \frac{25.5}{4} = 6.375$$

	$(4.25, 6.375)$	$(6.375, 8.5)$
sign of f'	$-$	$+$
f	↘	↗

We have global minimum at $\underline{x = 6.375}$

Corresponding $y = \sqrt{\dfrac{8.5x^2}{2x-8.5}} \approx \underline{9.0156} < 11$

(We need to check that $y < 11$ as we have not used that restriction.) Then the minimal $L = x^2+y^2 \approx \underline{\underline{11.042}}$