5. What substitutions are sometimes used to transform integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, and $\sqrt{x^2 - a^2}$ into integrals that can be evaluated directly? Give an example of each case.
6. What restrictions can you place on the variables involved in the three basic trigonometric substitutions to make sure the substitutions are reversible (have inverses)?
7. What is the goal of the method of partial fractions?
8. When the degree of a polynomial $f(x)$ is less than the degree of a polynomial $g(x)$, how do you write $f(x)/g(x)$ as a sum of partial fractions if $g(x)$
   a. is a product of distinct linear factors?
   b. consists of a repeated linear factor?
   c. contains an irreducible quadratic factor?
   What do you do if the degree of f is *not* less than the degree of g?
9. How are integral tables typically used? What do you do if a particular integral you want to evaluate is not listed in the table?
10. What is a reduction formula? How are reduction formulas used? Give an example.
11. How would you compare the relative merits of Simpson's Rule and the Trapezoidal Rule?
12. What is an improper integral of Type I? Type II? How are the values of various types of improper integrals defined? Give examples.
13. What tests are available for determining the convergence and divergence of improper integrals that cannot be evaluated directly? Give examples of their use.
14. What is a random variable? What is a continuous random variable? Give some specific examples.
15. What is a probability density function? What is the probability that a continuous random variable has a value in the interval $[c, d]$?
16. What is an exponentially decreasing probability density function? What are some typical events that might be modeled by this distribution? What do we mean when we say such distributions are *memoryless*?
17. What is the expected value of a continuous random variable? What is the expected value of an exponentially distributed random variable?
18. What is the median of a continuous random variable? What is the median of an exponential distribution?
19. What does the variance of a random variable measure? What is the standard deviation of a continuous random variable X?
20. What probability density function describes the normal distribution? What are some examples typically modeled by a normal distribution? How do we usually calculate probabilities for a normal distribution?
21. In a normal distribution, what percentage of the population lies within 1 standard deviation of the mean? Within 2 standard deviations?

Chapter 8 Practice Exercises

Integration by Parts

Evaluate the integrals in Exercises 1–8 using integration by parts.

1. $\int \ln (x+1)\, dx$
2. $\int x^2 \ln x\, dx$
3. $\int \tan^{-1} 3x\, dx$
4. $\int \cos^{-1}\left(\frac{x}{2}\right) dx$
5. $\int (x+1)^2 e^x\, dx$
6. $\int x^2 \sin (1-x)\, dx$
7. $\int e^x \cos 2x\, dx$
8. $\int x \sin x \cos x\, dx$

Partial Fractions

Evaluate the integrals in Exercises 9–28. It may be necessary to use a substitution first.

9. $\int \frac{x\, dx}{x^2 - 3x + 2}$
10. $\int \frac{x\, dx}{x^2 + 4x + 3}$
11. $\int \frac{dx}{x(x+1)^2}$
12. $\int \frac{x+1}{x^2(x-1)}\, dx$
13. $\int \frac{\sin \theta\, d\theta}{\cos^2\theta + \cos\theta - 2}$
14. $\int \frac{\cos \theta\, d\theta}{\sin^2\theta + \sin\theta - 6}$
15. $\int \frac{3x^2 + 4x + 4}{x^3 + x}\, dx$
16. $\int \frac{4x\, dx}{x^3 + 4x}$
17. $\int \frac{v+3}{2v^3 - 8v}\, dv$
18. $\int \frac{(3v-7)\, dv}{(v-1)(v-2)(v-3)}$
19. $\int \frac{dt}{t^4 + 4t^2 + 3}$
20. $\int \frac{t\, dt}{t^4 - t^2 - 2}$
21. $\int \frac{x^3 + x^2}{x^2 + x - 2}\, dx$
22. $\int \frac{x^3 + 1}{x^3 - x}\, dx$
23. $\int \frac{x^3 + 4x^2}{x^2 + 4x + 3}\, dx$
24. $\int \frac{2x^3 + x^2 - 21x + 24}{x^2 + 2x - 8}\, dx$
25. $\int \frac{dx}{x(3\sqrt{x+1})}$
26. $\int \frac{dx}{x\left(1 + \sqrt[3]{x}\right)}$
27. $\int \frac{ds}{e^s - 1}$
28. $\int \frac{ds}{\sqrt{e^s + 1}}$

Trigonometric Substitutions

Evaluate the integrals in Exercises 29–32 **(a)** without using a trigonometric substitution, **(b)** using a trigonometric substitution.

29. $\int \frac{y\, dy}{\sqrt{16 - y^2}}$
30. $\int \frac{x\, dx}{\sqrt{4 + x^2}}$
31. $\int \frac{x\, dx}{4 - x^2}$
32. $\int \frac{t\, dt}{\sqrt{4t^2 - 1}}$

Evaluate the integrals in Exercises 33–36.

33. $\int \frac{x\,dx}{9 - x^2}$ **34.** $\int \frac{dx}{x(9 - x^2)}$

35. $\int \frac{dx}{9 - x^2}$ **36.** $\int \frac{dx}{\sqrt{9 - x^2}}$

Trigonometric Integrals

Evaluate the integrals in Exercises 37–44.

37. $\int \sin^3 x \cos^4 x\,dx$ **38.** $\int \cos^5 x \sin^5 x\,dx$

39. $\int \tan^4 x \sec^2 x\,dx$ **40.** $\int \tan^3 x \sec^3 x\,dx$

41. $\int \sin 5\theta \cos 6\theta\,d\theta$ **42.** $\int \sec^2 \theta \sin^3 \theta\,d\theta$

43. $\int \sqrt{1 + \cos(t/2)}\,dt$ **44.** $\int e^t \sqrt{\tan^2 e^t + 1}\,dt$

Numerical Integration

45. According to the error-bound formula for Simpson's Rule, how many subintervals should you use to be sure of estimating the value of

$$\ln 3 = \int_1^3 \frac{1}{x}\,dx$$

by Simpson's Rule with an error of no more than 10^{-4} in absolute value? (Remember that for Simpson's Rule, the number of subintervals has to be even.)

46. A brief calculation shows that if $0 \le x \le 1$, then the second derivative of $f(x) = \sqrt{1 + x^4}$ lies between 0 and 8. Based on this, about how many subdivisions would you need to estimate the integral of f from 0 to 1 with an error no greater than 10^{-3} in absolute value using the Trapezoidal Rule?

47. A direct calculation shows that

$$\int_0^{\pi} 2\sin^2 x\,dx = \pi.$$

How close do you come to this value by using the Trapezoidal Rule with $n = 6$? Simpson's Rule with $n = 6$? Try them and find out.

48. You are planning to use Simpson's Rule to estimate the value of the integral

$$\int_1^2 f(x)\,dx$$

with an error magnitude less than 10^{-5}. You have determined that $|f^{(4)}(x)| \le 3$ throughout the interval of integration. How many subintervals should you use to ensure the required accuracy? (Remember that for Simpson's Rule the number has to be even.)

T **49. Mean temperature** Use Simpson's Rule to approximate the average value of the temperature function

$$f(x) = 37 \sin\left(\frac{2\pi}{365}(x - 101)\right) + 25$$

for a 365-day year. This is one way to estimate the annual mean air temperature in Fairbanks, Alaska. The National Weather Service's official figure, a numerical average of the daily normal mean air temperatures for the year, is 25.7°F, which is slightly higher than the average value of $f(x)$.

50. Heat capacity of a gas Heat capacity C_v is the amount of heat required to raise the temperature of a given mass of gas with constant volume by 1°C, measured in units of cal/deg-mol (calories per degree gram molecular weight). The heat capacity of oxygen depends on its temperature T and satisfies the formula

$$C_v = 8.27 + 10^{-5}(26T - 1.87T^2).$$

Use Simpson's Rule to find the average value of C_v and the temperature at which it is attained for $20° \le T \le 675°$C.

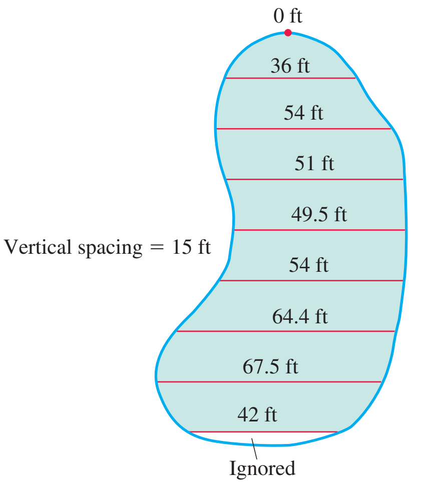
51. Fuel efficiency An automobile computer gives a digital readout of fuel consumption in gallons per hour. During a trip, a passenger recorded the fuel consumption every 5 min for a full hour of travel.

Time	Gal/h	Time	Gal/h
0	2.5	35	2.5
5	2.4	40	2.4
10	2.3	45	2.3
15	2.4	50	2.4
20	2.4	55	2.4
25	2.5	60	2.3
30	2.6		

a. Use the Trapezoidal Rule to approximate the total fuel consumption during the hour.

b. If the automobile covered 60 mi in the hour, what was its fuel efficiency (in miles per gallon) for that portion of the trip?

52. A new parking lot To meet the demand for parking, your town has allocated the area shown here. As the town engineer, you have been asked by the town council to find out if the lot can be built for $11,000. The cost to clear the land will be $0.10 a square foot, and the lot will cost $2.00 a square foot to pave. Use Simpson's Rule to find out if the job can be done for $11,000.



Improper Integrals

Evaluate the improper integrals in Exercises 53–62.

53. $\displaystyle\int_0^3 \frac{dx}{\sqrt{9-x^2}}$

54. $\displaystyle\int_0^1 \ln x\, dx$

55. $\displaystyle\int_0^2 \frac{dy}{(y-1)^{2/3}}$

56. $\displaystyle\int_{-2}^0 \frac{d\theta}{(\theta+1)^{3/5}}$

57. $\displaystyle\int_3^\infty \frac{2\, du}{u^2-2u}$

58. $\displaystyle\int_1^\infty \frac{3v-1}{4v^3-v^2}\, dv$

59. $\displaystyle\int_0^\infty x^2e^{-x}\, dx$

60. $\displaystyle\int_{-\infty}^0 xe^{3x}\, dx$

61. $\displaystyle\int_{-\infty}^\infty \frac{dx}{4x^2+9}$

62. $\displaystyle\int_{-\infty}^\infty \frac{4\, dx}{x^2+16}$

Which of the improper integrals in Exercises 63–68 converge and which diverge?

63. $\displaystyle\int_6^\infty \frac{d\theta}{\sqrt{\theta^2+1}}$

64. $\displaystyle\int_0^\infty e^{-u}\cos u\, du$

65. $\displaystyle\int_1^\infty \frac{\ln z}{z}\, dz$

66. $\displaystyle\int_1^\infty \frac{e^{-t}}{\sqrt{t}}\, dt$

67. $\displaystyle\int_{-\infty}^\infty \frac{2\, dx}{e^x+e^{-x}}$

68. $\displaystyle\int_{-\infty}^\infty \frac{dx}{x^2(1+e^x)}$

Assorted Integrations

Evaluate the integrals in Exercises 69–116. The integrals are listed in random order so you need to decide which integration technique to use.

69. $\displaystyle\int \frac{x\, dx}{1+\sqrt{x}}$

70. $\displaystyle\int \frac{x^3+2}{4-x^2}\, dx$

71. $\displaystyle\int \sqrt{2x-x^2}\, dx$

72. $\displaystyle\int \frac{dx}{\sqrt{-2x-x^2}}$

73. $\displaystyle\int \frac{2-\cos x+\sin x}{\sin^2 x}\, dx$

74. $\displaystyle\int \sin^2\theta\cos^5\theta\, d\theta$

75. $\displaystyle\int \frac{9\, dv}{81-v^4}$

76. $\displaystyle\int_2^\infty \frac{dx}{(x-1)^2}$

77. $\displaystyle\int \theta\cos(2\theta+1)\, d\theta$

78. $\displaystyle\int \frac{x^3\, dx}{x^2-2x+1}$

79. $\displaystyle\int \frac{\sin 2\theta\, d\theta}{(1+\cos 2\theta)^2}$

80. $\displaystyle\int_{\pi/4}^{\pi/2} \sqrt{1+\cos 4x}\, dx$

81. $\displaystyle\int \frac{x\, dx}{\sqrt{2-x}}$

82. $\displaystyle\int \frac{\sqrt{1-v^2}}{v^2}\, dv$

83. $\displaystyle\int \frac{dy}{y^2-2y+2}$

84. $\displaystyle\int \frac{x\, dx}{\sqrt{8-2x^2-x^4}}$

85. $\displaystyle\int \frac{z+1}{z^2(z^2+4)}\, dz$

86. $\displaystyle\int x^2(x-1)^{1/3}\, dx$

87. $\displaystyle\int \frac{t\, dt}{\sqrt{9-4t^2}}$

88. $\displaystyle\int \frac{\tan^{-1}x}{x^2}\, dx$

89. $\displaystyle\int \frac{e^t\, dt}{e^{2t}+3e^t+2}$

90. $\displaystyle\int \tan^3 t\, dt$

91. $\displaystyle\int_1^\infty \frac{\ln y}{y^3}\, dy$

92. $\displaystyle\int y^{3/2}(\ln y)^2\, dy$

93. $\displaystyle\int e^{\ln\sqrt{x}}\, dx$

94. $\displaystyle\int e^\theta\sqrt{3+4e^\theta}\, d\theta$

95. $\displaystyle\int \frac{\sin 5t\, dt}{1+(\cos 5t)^2}$

96. $\displaystyle\int \frac{dv}{\sqrt{e^{2v}-1}}$

97. $\displaystyle\int \frac{dr}{1+\sqrt{r}}$

98. $\displaystyle\int \frac{4x^3-20x}{x^4-10x^2+9}\, dx$

99. $\displaystyle\int \frac{x^3}{1+x^2}\, dx$

100. $\displaystyle\int \frac{x^2}{1+x^3}\, dx$

101. $\displaystyle\int \frac{1+x^2}{1+x^3}\, dx$

102. $\displaystyle\int \frac{1+x^2}{(1+x)^3}\, dx$

103. $\displaystyle\int \sqrt{x}\cdot\sqrt{1+\sqrt{x}}\, dx$

104. $\displaystyle\int \sqrt{1+\sqrt{1+x}}\, dx$

105. $\displaystyle\int \frac{1}{\sqrt{x}\cdot\sqrt{1+x}}\, dx$

106. $\displaystyle\int_0^{1/2} \sqrt{1+\sqrt{1-x^2}}\, dx$

107. $\displaystyle\int \frac{\ln x}{x+x\ln x}\, dx$

108. $\displaystyle\int \frac{1}{x\cdot\ln x\cdot\ln(\ln x)}\, dx$

109. $\displaystyle\int \frac{x^{\ln x}\ln x}{x}\, dx$

110. $\displaystyle\int (\ln x)^{\ln x}\left[\frac{1}{x}+\frac{\ln(\ln x)}{x}\right] dx$

111. $\displaystyle\int \frac{1}{x\sqrt{1-x^4}}\, dx$

112. $\displaystyle\int \frac{\sqrt{1-x}}{x}\, dx$

113. **a.** Show that $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$.

b. Use part (a) to evaluate

$$\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\, dx.$$

114. $\displaystyle\int \frac{\sin x}{\sin x+\cos x}\, dx$

115. $\displaystyle\int \frac{\sin^2 x}{1+\sin^2 x}\, dx$

116. $\displaystyle\int \frac{1-\cos x}{1+\cos x}\, dx$

Chapter 8 Additional and Advanced Exercises

Evaluating Integrals

Evaluate the integrals in Exercises 1–6.

1. $\displaystyle\int (\sin^{-1}x)^2\, dx$

2. $\displaystyle\int \frac{dx}{x(x+1)(x+2)\cdots(x+m)}$

3. $\displaystyle\int x\sin^{-1}x\, dx$

4. $\displaystyle\int \sin^{-1}\sqrt{y}\, dy$