Math 21B. GRANNER

These problems are from a Math 21A final. Solve them to refresh your memory. *Do not turn your solutions in.*

1. Compute the following limits, *in any correct way* you can.

(a) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + x} - \sqrt{x^2 - x} \right)$

(b) $\lim\limits_{x \to 0} \dfrac{\ln(1 + x) - x}{e^x - 1 - x}$

(c) $\lim\limits_{h \to 0} \dfrac{(2 + h)^5 - 32}{h}$

(d) $\lim\limits_{x \to \infty} \left(\dfrac{2x - 3}{2x - 4} \right)^{3x - 4}$

2.

(a) $f(x) = \arctan \sqrt{\sin x}$. Compute $f'(x)$. *Do not simplify.*

(b) $f(x) = x \cdot \arcsin(\ln x)$. Compute $f'(x)$. *Do not simplify.*

(c) Assume that $f(x)$ satisfies the equation $x \cdot f(x)^3 + x^3 + xe^{f(x)-1} = 12$ and that $f(2) = 1$. Compute $f'(2)$.

3. Roughly graph $y = \frac{1}{2}x^{3/2}$. Then find the point on the graph which is closest to $(2, 0)$. (Don't forget that it is enough to mazimize the *square* of the distance between $(2, 0)$ and a point (x, y) on the graph!)

4. Let $f(x) = \dfrac{x + 1}{x^2} = \dfrac{1}{x} + \dfrac{1}{x^2}$. Sketch the graph of $y = f(x)$ using the first and second derivative. Be sure to label clearly all important points on the graph.

What is the range of this function? Is this function one–to–one on the interval $(0, \infty)$? Explain.