Math 21B, Fall 2023.

Discussion Problems 3 (Thu., Oct. 12)

1. Find $F'(x)$ if

(a) $F(x) = \displaystyle\int_{\sqrt{x}}^{\pi} e^{2t^2}\,dt,$ (b) $F(x) = \displaystyle\int_{-x}^{x} \frac{1}{3+t^2}\,dt.$

2. Compute

$$\lim_{x\to 0} \frac{\int_2^{2+5x} e^{t^2}\,dt}{\int_1^{1+x} e^{-t^2}\,dt}.$$

3. Let $f(x) = x + 1/x$. For which interval $I = [a, a+2]$, $a > 0$, is the average of f over I minimal?

4. Compute:

(a) $\displaystyle\int \frac{x^2+1}{(x-1)^3}\,dx$ (b) $\displaystyle\int_2^3 \frac{x^2+1}{(x-1)^3}\,dx$ (c) $\displaystyle\int \frac{1}{(x^2+1)\arctan x}\,dx$

(d) $\displaystyle\int_0^{\pi/2} \frac{\sin\theta}{1+\cos^2\theta}\,d\theta$ (e) $\displaystyle\int_0^1 (x^2+1)^5 x^3\,dx$ (f) $\displaystyle\int_0^{\pi/2} \sin x\cdot\cos x\cdot\sqrt{1-\cos x}\,dx$