Math 21B, Fall 2023.

Discussion Problems 4 (Thu., Oct. 19)

1. Find the area of the planar region bounded by curve $y = e^x + 2$, and the lines $x = 0$, $x = 2$, and $y = 0$.

2. (a) The planar region R is bounded by the graph of $y = -x^2 + 4x - 2$ and the line $y = x$. Compute the area of R. (b) Now, R is bounded by the graph of $y = -x^2 + 4x + \sqrt{x^{17} + 1} + 2022$ and the graph of $y = x + \sqrt{x^{17} + 1} + 2024$. Compute the area of R. (c) Finally, R is bounded by the graph of $y = -x^2 + 4x - 2$ and the lines $x = 0$ and $x + y = 2$. Compute the area of R.

3. The planar region R is bounded by curves $y = \sqrt{x}$ and $x = y^3 - 2y^2$. Compute its area.

4. Compute $\displaystyle\int_{-3}^{3} x \cdot (\sqrt{x+3} + \sin(x^4) + \cos(x^3))\, dx$.

5. If $\displaystyle\int_{-1}^{2} f(x)\, dx = 3$ and $\displaystyle\int_{0}^{2} f(x)\, dx = -4$, what is $\displaystyle\int_{0}^{-1} f(x)\, dx$?

6. Say we want to compute the integral $\displaystyle\int_{-1}^{1} x^2 \sqrt{x+2}\, dx$. Our strategy is to rewrite the integral as

$$\int_{-1}^{1} x\sqrt{x+2} \cdot x\, dx,$$

and introduce the substitution $u = x^2$. We get $du = 2x\, dx$, $x = \sqrt{u}$, $\begin{array}{c|c} x & u \\ \hline -1 & 1 \\ 1 & 1 \end{array}$, which results in

$$\int_{-1}^{1} x\sqrt{x+2} \cdot x\, dx = \frac{1}{2}\int_{1}^{1} \sqrt{u}\sqrt{\sqrt{u}+2}\, du = 0.$$

Is this correct? If it is not, point out the error and compute the integral correctly.